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## OPTIMAL STRATEGY FOR WATER PROTECTION ACTIVITY IN REGIONS

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### Abstract

The methodology for constructing mathematical models to select the optimal strategy for water protection activity in the regions and for water consumers is considered. At the same time, the water environment is considered as a whole with all technical, environmental, economic and other problems associated with it.

Particular attention is paid to linking and optimizing investments in water protection measures with maximizing the growth rates of the regional economy in the conditions of a given dynamics of water pollution at the planned growth rates of the economy.

An analysis of mathematical models of the optimal strategy for water protection activities at the level of water consumer warnings is also given.

Keywords: modeling, strategy, region, water environment, water management activities, river basin, water consumers.

## ОСОБЕННОСТИ ОПТИМАЛЬНОЙ СТРАТЕГИИ ВОДООХРАННОЙ ДЕЯТЕЛЬНОСТИ В РЕГИОНАХ

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### Реферат

Рассмотрена методология построения математических моделей по выбору оптимальной стратегии водоохранной деятельности в регионах и для водопотребителей. При этом водная среда рассматривается как единое целое со всеми техническими, экологическими, экономическими и другими, связанными с ней, проблемами.

Особое внимание уделено увязке и оптимизации инвестиций на водоохранные мероприятия с максимизацией темпов роста экономики регионов при заданной динамике загрязнений водной среды при планируемых темпах роста экономики.

Приведен также анализ математических моделей оптимальной стратегии водоохранной деятельности и на уровне предупреждений – водопотребителей.

Ключевые слова: моделирование, стратегия, регион, водная среда, водохозяйственная деятельность, речной бассейн, водопотребители.

### Introduction

Today, the problem of optimal water use has become more urgent than ever before.

The new economic policy and modern requirements for the greening of industries and technologies require not so much the preservation of water resources in a state close to the current one, but the restoration of their natural potential.

Current changes in the rate of most natural and man-made processes have led to a violation of biological, geochemical, genetic, resource-raw material and many other types of natural equilibrium and to uncertainty of the state of the water environment, strategy and tactics of interaction between the population, production, economy and nature [1, 2, 3].

The exponentially increasing scale of the anthropogenic impact on the water environment, its negative consequences and the need to optimize this impact require an active search for ways to solve these problems.

## Substantiation of the structure of water protection activity models

The models of economic development of water regions that are common today usually a priori assume unlimited natural resources or do not take into account possible losses associated with violation of natural processes in the natural environment and the cost of preventing these violations through a set of water protection measures.

To understand the limits of the models being developed, it should be noted that the inclusion of economic characteristics in them affects the quality of information support, since such indicators within the objective functions and constraints of most optimization problems turn out to be deliberately the most inaccurate and uncertain information, even against the background of insufficiency or inadequacy of other data.

The uniqueness of water systems practically eliminates the possibility of an active experiment; therefore, the construction and use of appropriate mathematical models becomes important for predicting and assessing the state of water resources.

Depending on the spatial and temporal scales of anthropogenic impacts, models can be local, regional or global.

The basic version of the model structure for the selection of water protection strategies in the river basin (region) is shown in Fig. 1 [4].

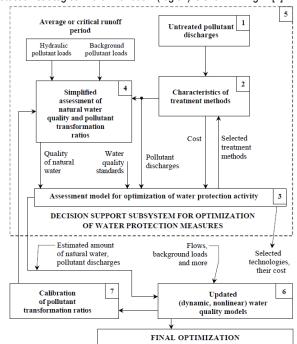


Figure 1 - Basic version of the structure of water protection models

Block 1 combines information on the composition, volumes and modes of pollutant discharges. Possible measures for treatment of these discharges are systematized in block 2. At the same time, for each method of wastewater treatment in the context of the considered pollutants or their groups, production functions are preliminarily compiled that characterize the relationship between the costs of carrying out the appropriate water protection measures and the degree of treatment.

The assessment of the quality of natural water should be based on aggregated equations for the transfer and transformation of pollutants and their transformation ratios, taking into account the total runoff of pollutants and the average anthropogenic load over a critical (usually lowwater) period of time. The estimated transformation ratios of pollutants in the river sections are also generated and the quality of natural water is assessed (block 4). Then, knowing the quality of natural water in the sections of the calculated areas, we can compare it with the requirements of the relevant standards using the assessment model for optimization of water protection activity (block 3), which allows us to select the recommended treatment technologies, permissible volumes of pollutant discharges and the required costs for the implementation of measures.

The set of mathematical models presented in blocks 1 - 4 (block 5) forms a system for substantiating decisions on the optimization of water protection activities. The search for optimal solutions at this stage is reduced to the choice of a set of measures that ensure the specified water quality at minimal cost.

However, such decisions can't always be taken as final, since they are obtained on the basis of approximate and incomplete information. Therefore, it is advisable to refine these solutions using models (block 6) that take into account the nonlinearity of dependencies characterizing water quality, dynamic relationships between many parameters, etc. As a rule, it is necessary to operate with time series of river runoff and discharges of pollutants, and the forecast of water quality comes to these models from the results of calculations according to the estimation model of optimization. These models also make it possible to refine the transformation ratios of different substances by sections of the river (block 7). If there are significant discrepancies between them and the estimated values of the coefficients obtained by implementing the simplified models of blocks 4 and 5, it is advisable to return to the assessment model with refined natural water quality indicators, that is, an iterative process of applying assessment models and more detailed models arises

Decision-making at the water user level can be represented by models in which, under various restrictions in the conditions of the economic mechanism of pollution charges, the objective function is optimized, which includes not only the costs of these measures, but also changing water pollution charges.

## Choosing the optimal strategy

When choosing the optimal strategy for water protection activities both in the region and for individual water use, in the retrospective period, the determining factor is the relationship and interdependence of environmental and economic indicators and criteria [5, 6, 7].

There are a number of studies that propose approaches to assess the ecological state of the water environment depending on the factors determining the economic development of the regions and their environmental policy [8, 9].

The most common approach is based on pollution functions, which allow us to analyze various options for the distribution of investment, assess the impact of changes in the structure of the economy and take into account the impact of environmental policy by considering the dynamics of investments and costs associated with the protection of the water environment.

$$E(t) = A(t) \cdot X_1^{\mu}(t) \cdot X_2^{-\eta}(t), \tag{1}$$

$$E(t) = A(t) \cdot X_1^{\mu}(t) \cdot X_2^{-\eta}(t) \cdot X_3^{\nu}(t), \tag{2}$$

where E(t) is the environmental indicator under study (pollutant emissions, pollutant concentration, polluted wastewater discharge, waste generation and other indicators);  $X_1(t)$  is a factor that reflects the development of the economy and, as a rule, negatively affects the water environment (investment in manufacturing sector, investment in building construction and other factors);  $X_2(t)$  is a factor that reflects water protection activities and has a positive impact on the environment (investment in environmental protection, current costs for environmental protection and other factors);  $X_3(t)$  is a factor that reflects the development of the economy and can affect both positively and negatively on the environment, depending on the environmental and economic policy being implemented (investment in modernization of the economy, in machinery and equipment, the index of structural changes in the economy and other factors); A(t) is neutral environmental progress (decrease in the level of pollution due to other factors, primarily structural changes);  $\mu$ ,  $\eta$ ,  $\nu$  are constant parameters (factor elasticities); t is time.

Since these functions use investment indicators, the main goal is to find the optimal distribution of investment under restrictions on the total level.

At the same time, the optimal distribution of investment should be linked to the maximization of the economic growth rate at a given dynamics of pollution or, conversely, to the minimization of pollution at a given growth rate of the economy [10, 11, 12].

Optimization models of two types can be considered for three-factor environmental investment functions (Z<sub>3</sub>) and two-factor pollution functions (Z<sub>3</sub>). In the first case, when investment is divided into three components (new buildings construction, modernization of the economy, water environment protection), the problem of optimal distribution of investment in three areas arises. The optimal solution minimizes pollution while ensuring a certain volume of production  $Y_0(t)$ .

$$E(t) = F(X_1(t), X_2(t), X_3(t), t) \to min,$$
 (3)

subject to

$$\begin{split} Y(t) &= G\big(X_1(t), X_3(t)\big) \geq Y_0(t), \\ U(t) &= X_1(t) + X_2(t) + X_3(t), \\ X_1(t) \geq 0, X_2(t) \geq 0, \quad X_3(t) \geq 0, \quad \varepsilon_1 > 0, \quad \varepsilon_2 > 0, \end{split}$$

where Y(t) is a constraint on production volume growth; U(t) is a constraint on the level of investment; G is a function of economic growth depending on the level of investment in the relevant areas.

It should be noted that the directions of investment distribution under study have different effectiveness in terms of minimizing pollution. Investments in new buildings tends to increase pollution. Modernization most often slightly reduces the level of pollution. Water protection activities are most effective and always lead to a reduction in pollution.

In the absence of a constraint on the growth of production volume or its rate, all resources are directed to the most effective area in terms of the criterion that has the maximum modulo negative value of the parameter obtained by formula (water protection activities or modernization). If there is a constraint on the growth of production or its rate, depending on the volume of construction of new buildings and modernization, then the solution is based on the following procedure.

The first case is the impact of modernization on pollution is negative (parameter v is negative). Then the maximum investment is made in modernization, and if the desired economic growth rate is achieved, then the rest of the investments are directed to the protection of the water environment or the construction of new buildings, depending on the value of the corresponding parameter. If the given rate cannot be achieved by investing only in modernization, then the ratio of investment in the construction of new buildings and modernization is determined to ensure its achievement.

The second case - modernization leads to an increase in pollution (the parameter v is positive). If, with maximum investment in modernization, the given economic growth rate is achievable, then the remaining resources should be directed to the protection of the water environment and nature as a whole. If the given growth rate of the economy is not achievable, then the ratio of investment in the construction of new buildings and the modernization of the economy is determined, ensuring the achievement of the desired growth rate.

For a two-factor function, the optimal distribution minimizes pollution:

$$E(t)=\sum_{i}E_{i}(t)=\sum_{i}F_{i}\left(X_{1,i}(t),XU_{2,i}(t),t\right)\rightarrow min, \qquad \text{(4)}$$
 subject to

$$X_{1}(t) = \sum_{i} X_{1,i}(t), X_{2}(t) = \sum_{i} X_{2,i}(t),$$
  $X_{1,i}(t) \geq 0, \ X_{2,i}(t) \geq 0, \ X_{1,i}(t) \geq 0, \ \varepsilon_{1,i} > 0, \ \varepsilon_{2,i} \leq 0, \ i = \overline{1,N},$  where  $i$  is a sector.  $N$  is the number of sectors.

This model can be built for two-factor and three-factor functions. It is also possible to find an optimal solution in general and under certain constraints [13].

If the region's economy reaches a high enough level, its development can be described by the Kuznets environmental curve, when the volume of pollution decreases with the growth of investment. If both factor elasticities are negative, then constructing Lagrangian on the basis of (1) will result in optimal resource allocation conditions:

The optimizar resource anotocation conditions:
$$\frac{\varepsilon_{1i} \cdot E_i(t)}{X_{1i}} = \frac{\varepsilon_{1i} \cdot E_j(t)}{X_{1j}}, \quad \overline{\iota, j} = 1, \overline{N},$$

$$\frac{\varepsilon_{2i} \cdot E_i(t)}{X_{2i}} = \frac{\varepsilon_{2i} \cdot E_j(t)}{X_{2j}}, \quad \overline{\iota, j} = 1, \overline{N},$$
(5)

where i, j are sectors, N is the number of sectors according to the degree and nature of water environment pollution.

By substituting in (5) multiplicative functions (1) and balance equations from (4), we can obtain a system of two nonlinear equations with two variables, which can be easily solved by standard methods. If we accept that the sum of the factor elasticities of the sectors is the same (it is possible to introduce such a constraint when calculating the functions), the system is converted into an equation with respect to the ratio of the two factors. As a result, optimal resource allocation is achieved by sequential solving of nonlinear equations. For other types of functions, as well as for more complex criteria, a system of nonlinear equations is obtained [14, 15, 16].

For functions (5), cumulative investments or production volumes are considered, which complicates the transfer of resources (investments spent a few years ago are not transferable). Therefore, we can go to increment functions.

An optimal solution can be found in the general case, when investments can be distributed both by directions and by sectors:

$$E(t)=\sum_i E_i(t)=\sum_i F_i\left(X_{1,i}(t),X_{2,i}(t),X_{3,i}(t)\right)\to min, \ (6)$$
 subject to

$$X_1(t) = \sum_i X_{1,i}(t), \quad X_2(t) = \sum_i X_{2,i}(t), \quad X_3(t) = \sum_i X_{3,i}(t),$$
$$Y(t) = G(X_1(t), X_3(t)) \ge Y_0(t),$$

$$X_{1,i}(t) \ge 0, X_{2,i}(t) \ge 0, \ X_{3,i}(t) \ge 0, \ \varepsilon_{1,i} > 0, \ \varepsilon_{2,i} \le 0, \ i = \overline{1,N}.$$

No less significant is the problem of finding ways to reduce pollutant discharges by water consumers. Therefore, the water protection strategy should be formulated in the form of an optimization problem with a criterion for minimizing total costs, subject to the achievement of a set of required standards for the quality of water resources and taking into account restrictions on the concentration and mass of pollutant discharges. The system of equations has the form [15, 17]:

$$S = \sum_{i \in I} \sum_{t \in T_i} (S_{it} \cdot x_{it}) \to \min;$$
 (7)

subject to

$$C_{jr} = C_{jr}^{0} + \sum_{i \in I} \sum_{t \in T_{i}} \left( m_{ijt} \cdot \theta_{ir} \cdot x_{it} \right) \leq \overline{C}_{jr}, \quad j \in J, \quad r \in R, \quad (8)$$

$$M_{jr} = \sum_{i \in T} \sum_{t \in T_{i}} \left( m_{ijt} \cdot x_{it} \right) \leq \overline{M}_{jr}, \quad M_{j} = \sum_{i \in T} \sum_{t \in T_{i}} \left( m_{ijt} \cdot x_{it} \right) \leq \overline{M}_{j}, \quad j \in J, \quad r \in R, \quad (9)$$

where  $M_{jr}$ ,  $M_{j}$  are, respectively, the total masses of pollutant discharges within individual sectors and within the entire basin;  $C_{jr}$  is the maximum allowable concentration of the i-th component;  $m_{ijt}$  is the mass of the minimum possible discharge corresponding to the treatment technology with the maximum cost  $S_{ij}$ .

It should be noted that, in general, there are no special guarantees for improving the efficiency of water protection activities only through the use of economic mechanisms. Moreover, these mechanisms do not exclude the traditional planning of a strategy for improving the quality of natural waters. Economic mechanisms can be only one of the important components, allowing to increase the profitability of water protection measures.

The simplest way to reflect economic mechanisms in mathematical models is the introduction of penalty payments into the objective function. Then the problem of optimization of water protection activities is formulated as:

$$\hat{S} = \sum_{i \in I} \sum_{t \in T} (S_{it} \cdot x_{it}) + \sum_{r \in R} \sum_{j \in J} (\Delta c_{jr} \cdot p_{jr}) \to min, \quad (10)$$
 subject to

subject to 
$$c_{jr} = c_{jr}^0 + \sum_{i \in I} \sum_{t \in T} \left( m_{ijt} \cdot \theta_{ir} \cdot x_{it} \right), \Delta c_{jr} = max \left( 0; \ c_{jr} - \bar{C}_j \right), \text{(11)}$$
 
$$j \in J, \ r \in R; \qquad c_{jr} = \Delta c_{jr} \leq \bar{C}_{jr},$$
 where  $\Delta c_{jr}$  is a possible violation of the maximum allowable concentra-

where  $\Delta c_{jr}$  is a possible violation of the maximum allowable concentration (value  $\bar{C}_i$ ) for the *j*-th pollutant in the *r* section.

#### Conclusion

The intensification of economic activity and the corresponding technological complication of the schemes of operation of water management facilities and systems have now reached such a level, that the deterministic description of the causal relationship of all processes proved to be inconsistent. There is always an element of randomness that often leads to undesirable, including catastrophic situations.

Analysis of the presented models shows that the basic version of the internal structure of the mathematical model system for choosing the optimal water protection strategy in the river basin should generally include:

- models of water management mechanisms;
- objective functions of the parties involved in the process;
- production functions of various types of water protection activities, taking into account the relationship of treatment costs with the degree of purification of pollutants.

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