

## COMPARATIVE ANALYSIS OF THE CALCULATION OF THE STRESS INTENSITY FACTOR FROM THE RESULTS OF EQUILIBRIUM AND NON-EQUILIBRIUM TESTS

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### Abstract

This article explores the practical use of methods for determining the stress intensity factor at normal separation: eccentric compression of notched cubes and four-point bending of a notched beam. During non-equilibrium tests, the stress intensity factor value was calculated from the value of the breaking load. During equilibrium tests, the stress intensity factor value was determined from the complete equilibrium deformation diagram, taking into account the energy indicators of destruction. The test used nanofiber-reinforced concrete, in which carbon nanotubes are used as crack propagation inhibitors at the level of the cementing agent, and various macro-sized fibers are used at the level of fine-grained concrete. As a result of the tests, it was found that the methods for determining the recovery factor from cubes with a notch and from deformation diagrams showed a good degree of convergence. Fiber reinforcement affects the fracture toughness of a nanocement composite, and high-modulus fiber has a greater effect on fracture toughness than low-modulus fiber. The stress intensity factor is a good indicator for comparing different types of fiber reinforcement in terms of their effect on fracture toughness.

**Keywords:** nanofibre-reinforced concrete, crack resistance, fracture toughness, stress intensity factor, nanotubes, dispersed reinforcement, deformation diagram, energy consumption.

### СРАВНИТЕЛЬНЫЙ АНАЛИЗ РАСЧЕТА КОЭФФИЦИЕНТА ИНТЕНСИВНОСТИ НАПРЯЖЕНИЙ ПО РЕЗУЛЬТАТАМ РАВНОВЕСНЫХ И НЕРАВНОВЕСНЫХ ИСПЫТАНИЙ

**Е. А. Садовская, С. Н. Леонович**

### Реферат

В данной статье исследовано практическое использование методов определения коэффициента интенсивности напряжений (КИН) при нормальном отрыве: внецентренное сжатие кубов с надрезами и четырехточечный изгиб балки с надрезом. При неравновесных испытаниях значение КИН рассчитывалось по величине разрушающей нагрузки. При равновесных испытаниях величина КИН определялась из полной равновесной диаграммы деформирования с учетом энергетических показателей разрушения. В испытании использовался нанофибробетон, в котором на уровне цементующего вещества в качестве ингибиторов распространения трещин используются углеродные нанотрубки, а на уровне мелкозернистого бетона – различные фибровые волокна макроразмера. В результате испытаний установлено, что методы определения КИН по кубам с надрезом и по диаграммам деформирования показали хорошую степень сходимости. Фибровое армирование оказывает влияние на вязкость разрушения наноцементного композита, причем высокомодульная фибра оказывает большее влияние по показателю вязкости разрушения, чем низко модульная. Коэффициент интенсивности напряжений является хорошим показателем для сравнения разных типов фибрового армирования по их влиянию на вязкость разрушения.

**Ключевые слова:** нанофибробетон, трещиностойкость, вязкость разрушения, коэффициент интенсивности напряжений, нанотрубки, дисперсное армирование, диаграмма деформирования, энергозатраты.

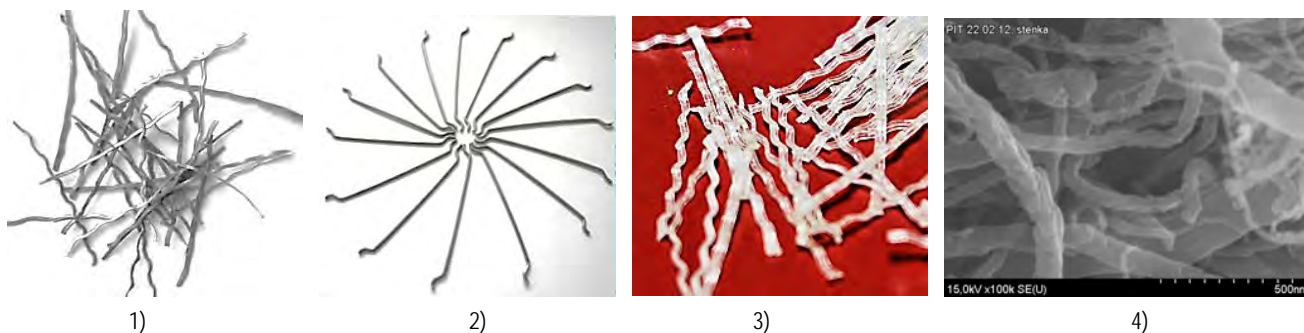
### Introduction

Nanofibre-reinforced concrete, from the point of view of a multilevel system [1], is a concrete composite with crack propagation inhibitors at the level of cementing agent and fine-grained concrete. As inhibitors, carbon nanotubes [2–4] and various macrosized fiber fibers (Fig. 1) [5, 6] are considered.

One of the distinguishing features of dispersed reinforced concrete is an increased crack resistance index [7]. Crack resistance (fracture toughness) is characterized by the magnitude of the stress intensity factor (SIF). The existence of many calculation and practical methods for

determining the recovery SIF [8–11], as well as the regular appearance of new ones, indicates difficulties in implementation and the presence of inaccuracies in their use.

The aim of the study is to develop a reliable method for calculating the stress intensity factor at normal separation of structural nanofibre-reinforced concrete based on the results of equilibrium and non-equilibrium tests.



1 – from sheet steel of a wave profile (FLV-0.9-50); 2 – made of steel wire with anchors (FPA-1.0); 3 – polymeric wavy (FPV-0.6-40); 4 – carbon nanotubes [6]

Figure 1 – Fiber

**Materials and research methods**

For the study, the following types of materials were used: Portland cement 500D20 JSC "Krasnoselskstroyaterialy"; construction sand (I class); rubble granite (III gr.); sulfoaluminate additive (RSAM), compacted condensed silica fume (MKU-85); chemical additive Relamix PC; nanomodified chemical additive ART-Konkrit R (aqueous suspension of nanostructured carbon (0.01-20 microns) and plasticizer).

The samples were made from the compositions of nanoconcrete mixtures A, Б, В, Г (table 1) with the addition of various types and amounts of dispersed fibers: Ф1 - wave steel fiber from a sheet (80 kg); Ф2 - steel wire anchor fiber (80 kg); Ф3 - wavy polymer fiber (4 kg).

**Table 1 – Formulations of the studied compositions**

Compositions	Cement	RSAM / MKU-85	Rubble granite 5-20 mm	Rubble granite 5-10 mm	Sand	Nanomodified chemical additive (% by weight of binder)
A	400	-	1020	-	820	3.2 (0.8)
Б	445	-	1035	-	820	2.22 (0.5)
В	460	-	-	880	950	3.22 (0.7)
Г	485	40/45	-	825	800	4.65 (0.7)

Normal separation on cubes with a notch. For tests, cube samples 100x100x100 mm were used with notches in the form of symmetrical notches with a depth of  $h / 4$  (where  $h$  is the height of the cube) made using diamond-coated cutting tools. Tests are carried out with eccentric compression (Fig. 2). Loading is carried out until the moment of separation of the sample into two parts or the formation of a crack, and the value of the destruction  $F_{IC}$  is recorded.

The value of the critical stress intensity factor for normal separation is calculated by the formula:

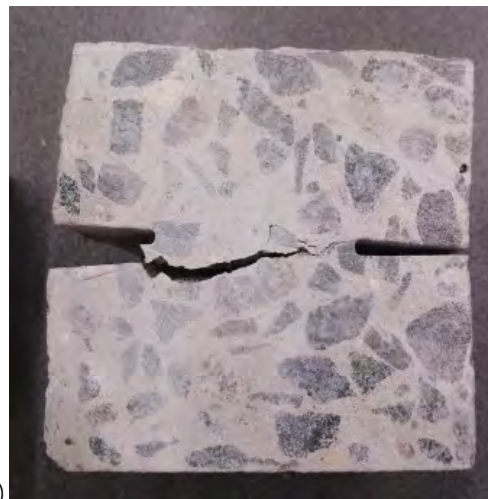
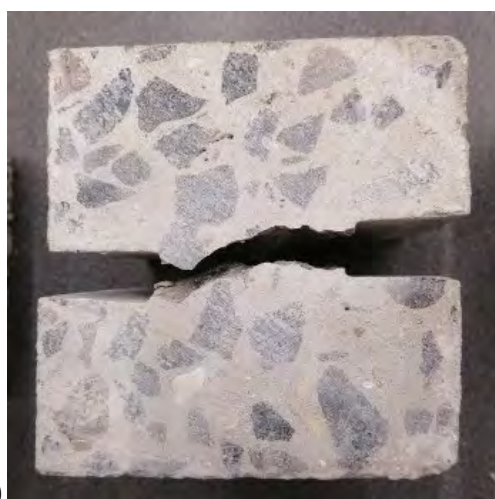
$$K_{c}^{*} = \frac{F_{ic}}{b \cdot h^{1/2}} \left[ \begin{matrix} 18,3 \left(\frac{a}{h}\right)^{1/2} - 430 \left(\frac{a}{h}\right)^{3/2} + 3445 \left(\frac{a}{h}\right)^{5/2} - \\ - 11076 \left(\frac{a}{h}\right)^{7/2} + 12967 \left(\frac{a}{h}\right)^{9/2} \end{matrix} \right], \quad (1)$$

where  $F_{IC}$  is the load at which failure occurs, in MN;

$b$  is the sample width, m;

$h$  is the sample height, m;

$a$  is the notch depth, m,  $a = h/4$ .



1 – appearance of the test, 2 – halves of the sample after testing, 3 – tested concrete sample, 4 – tested nanofibre-reinforced concrete sample

**Figure 2 – Normal pull test on notched cubes**

Normal separation when bending beams. Prism specimens 100x100x400 with a notch in the middle third were tested for tensile bending according to a four-point loading scheme with fixation of the complete equilibrium fracture diagram [12] (Fig. 3).

Static critical stress intensity factor [GOST 29167]

$$K_i = \sqrt{G_i E_b} \quad (2)$$

where  $G_i$  – the specific energy consumption for static fracture up to the moment when the main crack begins to move  $J/m^2$ ;

$E_b$  – the initial modulus of elasticity of nanofibre-reinforced concrete (GPa), determined by the formula (SP 52-104-2006):

$$E_{fb} = E_b(1 - \mu_{fv}) + \mu_f E_f \quad (4)$$

where  $E_b$  – the modulus of elasticity of concrete, the normative one is adopted here;

$E_f$  – the modulus of elasticity of the fiber;

$\mu_f$  – fiber reinforcement coefficient by volume.

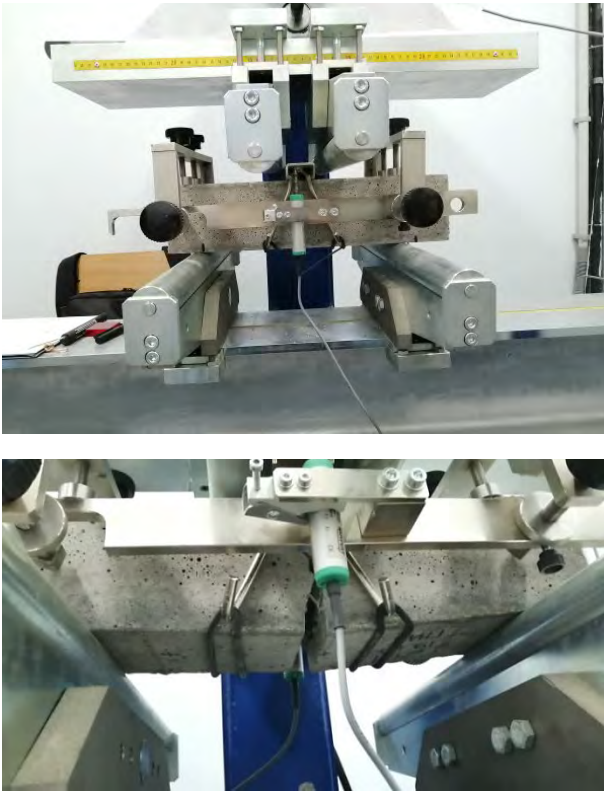


Figure 3 – Tensile testing of beam specimens in bending

In the nanofibre-reinforced concrete material, a pronounced plastic nature of tensile work is observed after the onset of cracking. In this case, the manifestation of the so-called deformation quasi-hardening can be achieved, which is characterized by the fact that after the onset of cracking, the stage of plastic tensile work of the material follows. In this case, the perceived stresses may exceed the stresses that cause cracking.

Table 2 – Parameters of nanofibre-reinforced concrete from equilibrium strain diagrams

Compositions	Max. load, kN $F$	Reduced strength, MPa $f^* = \frac{F \cdot l}{b(h-a)^2}$	Deflection at max. load, mm $f_{max}$	Specific energy consumption, $J/m^2$ $G_i = W/A_c$	Critical stress intensity factor, $MPa\sqrt{m}$		Relative deviation from the mean
					$K_i$	$K_c$	
A-Φ1	14.404	6.82	0.501	144.34	2.7	3.37	11%
Б-Φ1	14.97	6.91	0.200	26.56	1.0	1.01	0%
В-Φ1	16.011	7.58	0.671	91.75	1.8	2.05	7%
Г-Φ1	18.211	8.01	0.051	39.86	1.4	1.61	6%
A-Φ2	23.627	11.19	0.731	53.76	1.4	2.60	29%
Б-Φ2	16.207	7.67	0.671	82.28	1.8	2.24	11%
В-Φ2	16.313	7.72	0.200	55.12	1.4	1.32	2%
Г-Φ2	25.293	11.98	0.325	68.85	1.6	1.97	10%
A-Φ3	15.613	7.39	0.671	17.19	0.8	0.97	10%
Б-Φ3	15.111	7.15	0.055	37.59	1.17	0.99	8%
В-Φ3	11.121	5.27	0.051	21.04	0.8	0.69	9%
Г-Φ3	15.951	7.55	0.055	33.44	1.3	1.37	1%

If there is a zone of quasi-hardening in the diagram after the appearance of the first crack, the deformations do not concentrate in this one crack [12]. The material retains the ability to distribute cracks along the length of the stretched zone of the sample, while the cracks remain very small opening. This is ensured by the fact that the fiber distributed throughout the entire volume, with a sufficient modulus of elasticity of its material, strength and embedding in the matrix, completely perceives the tension from the concrete matrix in the cavity of the nucleated crack, not allowing it to increase sharply. The absence of manifestations of brittleness in tensile work makes it possible to take sufficiently large values of the material's resistance to tension in strength calculations. During design, this makes the balance of design checks of strength and crack resistance similar to that characteristic of reinforced concrete, i.e. with conventional bar reinforcement.

Based on the data obtained, a graph of Deflection-Time and Load-Deflection is constructed. Since the energy for starting the main crack (the sum of the elastic energy and the energy of microcrack formation), being the area under the curve, can increase tenfold depending on the fracture point. The start of the main crack when testing unreinforced concrete, as a rule, coincides with the moment of sample failure. To determine the start time of the main crack when testing nanofibre-reinforced concrete beams, an expert assessment of the researcher is required (Fig. 4).

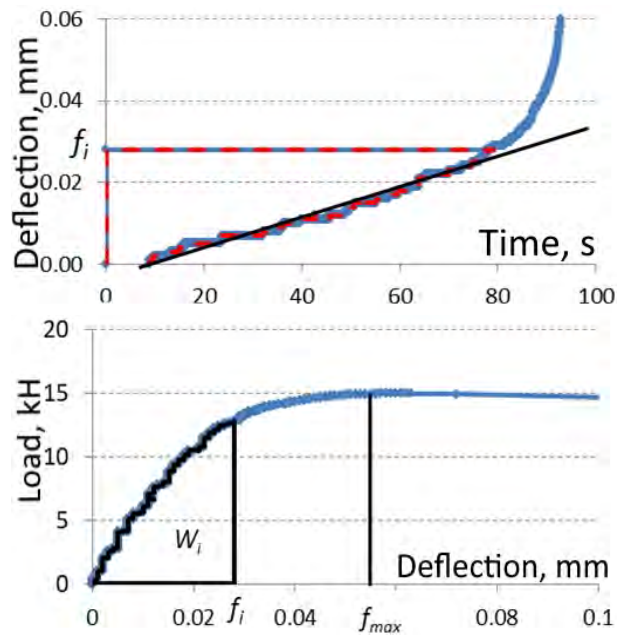


Figure 4 – Graphs of deformation

Analyzing the obtained deformation diagrams, one can obtain some important parameters characterizing the quality of the material under study: tensile strength in bending, deflection at maximum load ( $Z_{max}$ ), specific energy consumption for static fracture until the main crack begins to move ( $G_i$ ) (Table 2).

The obtained values of the stress intensity factor during testing by the method of normal separation on cubes with a notch ( $K_c$ ), when testing for four-point bending of beams with a notch ( $K_i$ ) and the average indicator ( $K$ ) are shown in Figure 5.

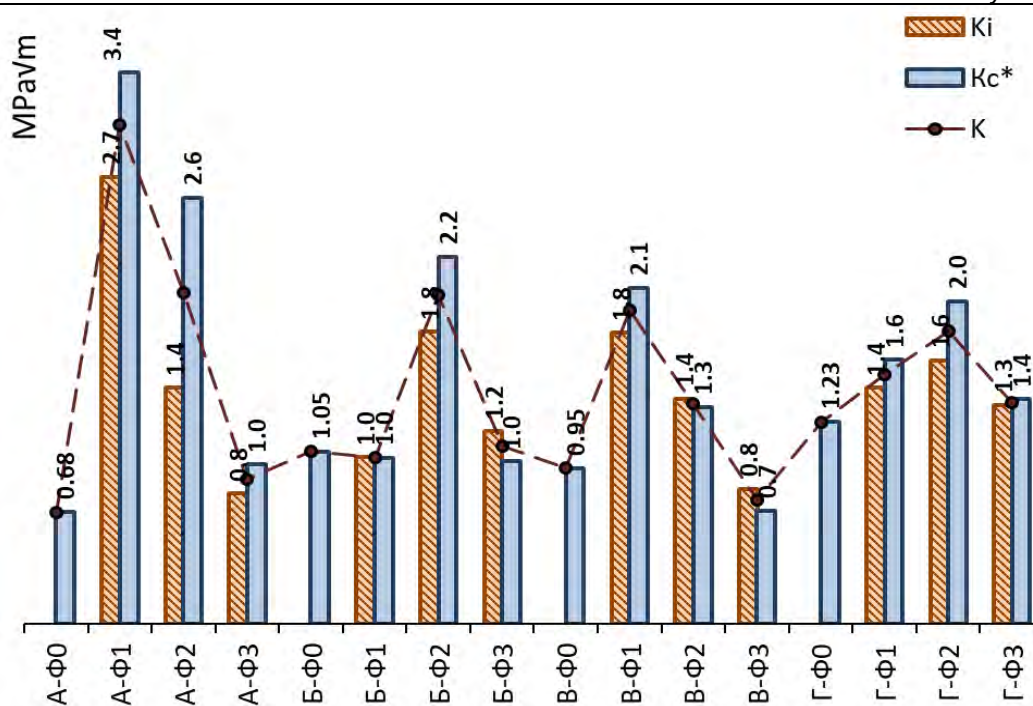


Figure 5 – Stress intensity factor for nanofibre-reinforced concrete

### Analysis of results

Good convergence of test results by the test methods used is observed. The trend of change in the fracture toughness index obtained by different test methods has the same character.

In compositions A and B, the high-modulus steel fiber (Φ1 and Φ2) has a greater effect on the fracture toughness index than the low-modulus one (Φ3).

In compositions B and Γ, steel wire fiber (Φ2) had the greatest effect on the fracture toughness index.

In all compositions, the effect of polymer fiber on the SIF value is the least and in some cases the value is close to unreinforced compositions. Fiber reinforcement with steel sheet fiber (Φ1) gives less stable fracture toughness than with steel wire reinforcement (Φ2).

### Conclusions

1. Methods for determining the oil recovery factor from cubes with a notch and from deformation diagrams showed a good degree of convergence.
2. The stress intensity factor is a good indicator for comparing different types of fiber reinforcement in terms of their effect on fracture toughness.
3. High modulus fiber has a greater impact on fracture toughness than low modulus.
4. Fiber reinforcement affects the fracture toughness of the nanocement composite.

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