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ESTIMATION OF THE SHEAR RESISTANCE MODELS OF REINFORCED CONCRETE ELEMENTS WITHOUT STIRRUPS ACCORDING TO VARIOUS DESIGN CODES

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Abstract
This paper presents the results of assessments shear design models with experimental data, included in the current and developed standards for the design of reinforced concrete structures.

Keywords: reinforced concrete, shear resistance, beams, estimation, design models, order statistics, confidence level, 5%-quantile.

ОЦЕНКА ПРОЧНОСТИ СРЕЗУ МОДЕЛЕЙ ЖЕЛЕЗОБЕТОННЫХ БАЛОЧНЫХ ЭЛЕМЕНТОВ БЕЗ ПОПЕРЕЧНОГО АРМИРОВАНИЯ ПО РАЗЛИЧНЫМ ПРОЕКТНЫМ КОДАМ

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Реферат
В данной статье представлены результаты оценок расчетных моделей на сдвиг с экспериментальными данными, включенные в действующие и разрабатываемые стандарты на проектирование железобетонных конструкций.

Ключевые слова: железобетон, сопротивление сдвигу, балки, оценка, расчетные модели, статистика заказов, уровень достоверности, 5% -квантиль.

Introduction

As shown in [10], the application of Eurocodes allows to development of a common understanding of the design problem and provides, on the one hand, the applying of harmonized design strategies for European countries, and on the other hand, opens up broad opportunities for international cooperation.

With the influx of a new generation of engineers in the countries of the united Europe and considering the fact that in the overwhelming majority of countries, national standards do not receive further development (funding for the development of national-level regulatory documents and research carried out for the purpose of normalization has been discontinued), in the design practice of Europe. There is practically no alternative to Eurocodes. But here, the absence of an alternative with broad harmonization creates serious problems. So, according to the current strategy in European standardization, the second generation of Eurocodes (EC - G2) was to be introduced in 2020. The fib Model Code 2010 forms the basis for the new reinforced concrete code. However, despite a rather extensive version of a new fib MC 2010, developed to replace MC 90, the new code for the design of reinforced concrete and pre-stressed structures (prEN 1992-1-1) was not accepted and implemented in 2020. Judging by the report of the chairman of the TG4 / TC250 working group on EC2 A. Muttoni, made in November 2019 at the 26th – Concrete Days (Czech Republic), the introduction of these codes may not take place by 2024.

One of the most open to question due to which a consensus among the scientific community has not been reached is still the problem of shear, including local shear (punching shear). So, according to [11], only based on the analysis of the results of the application of EN1992 (EC2), 1168 remarks and comments related to shear resistance models were collected. At the same time, until now, the thematic group TG4 / TC250 cannot choose for one of the considered variants of the shear resistance model and, accordingly, the local shear (punching shear).

Shear resistance models of elements without stirrups: a brief review

We accepted the following design models of the shear resistance of elements without stirrups for the analysis (see table 1):

1. The shear resistance design model according to the actual EC 2;
2. The shear design model according to ***fib*** Model Code 2010 (for two levels of approximation LoA I and LoA II) based on the Modified Compression Field Theory (MCFT) and Critical Shear Crack Theory (CSCT), that was recommended to introduce in a new version of the EC2. This model largely strives to get closer to understanding the physical phenomenon of the shear;
3. Semi-empirical shear resistance model based on the Critical Shear Crack Theory (CSCT), introduced in the prEN 1992 project.

Not so long ago, at conferences and seminars at various levels, passionate debates took place, in which the following issues were considered: for example, which model of resistance in bending, shear, punching shear is adequate, makes it possible to better describe the physical behavior of a structural element under load, etc.

As a rule, in the process of discussion, the results of verification of the proposed model against the background of experimental data obtained both in their own research and by various researchers are cited as an argument.

Let us briefly explain this using the example of the design models for calculating the shear resistance of elements without stirrups introduced in *fib* MC2010 [5] and prEC 2 [6] (see table 1). The design model equations are such that they consider one basic variable $\sqrt[3]{f_{ck}}$ or $\sqrt{f_{ck}}$, which expresses the characteristic shear strength of concrete as a function of the characteristic compression strength. The transition to the design values performs by dividing the characteristic values of the shear resistance by a partial coefficient $\gamma_c = 1{,}5$. In this situation, it should be noted that, ideally, the ratio $V_{theo} / V_{test} \cong 1$ is in the position corresponding to the 5% quantile of the distribution of the ratios of theoretical and experimental resistance, not the average value. Obviously, in this case, the average value must be a priori higher than 1.

It should be borne in mind that determining the position of the 5% quantile from the ratio of calculated and experimental values is also associated with certain problems. First of all, the estimation accuracy is due to the reasonable choice of the probability distribution function for the obtained empirical sample. As a rule, highly asymmetric distributions are obtained, for which the required quantile must still be calculated accordingly. In these cases, it can be very useful to use the method of order statistics [2], which was used in our analysis.

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Table 1 – Shear resistance models of RC- elements without stirrups

Codes	Design equations	Note
EC 2 [1]	$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \right] \cdot b_w \cdot d,$ but not less $V_{Rd,c} = (\nu_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d,$ $\nu_{min} = 0,035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$ if $0,5 \cdot d \le a_v < 2 \cdot d$ the value V_{Ed} is reduced by the coefficient $\beta = \frac{a_v}{2 \cdot d}$ $V_{Ed} \le 0,5 \cdot b_w \cdot d \cdot \nu \cdot f_{cd},$	$C_{Rd,c} = \frac{0,18}{\gamma_c};$ $k = 1 + \sqrt{\frac{200}{d}} \le 2,0;$ $\rho_l = \frac{A_{sl}}{b_w \cdot d} \le 0,02;$ $\nu = 0,6 \cdot \left(1 - \frac{f_{ck}}{250}\right).$
fib Model Code 2010 (LoA I) [5]	$V_{Rd,c} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot z \cdot b_w,$ $k_v = \frac{180}{1000 + 1,25 \cdot z},$ if $d \le a_v < 2 \cdot d$ the value V_{Ed} is reduced by the coefficient $\beta = \frac{a_v}{2 \cdot d}$	–
fib Model Code 2010 (LoA II) [5]	$V_{Rd,c} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot z \cdot b_w,$ $k_v = \frac{0,40}{1 + 1500 \cdot \varepsilon_x} \cdot \frac{1300}{1000 + k_{dg} \cdot z},$ $\varepsilon_x = \frac{1}{2 \cdot E_s \cdot A_s} \cdot \left(\frac{M_{Ed}}{z} + V_{Ed} + N_{Ed} \left(\frac{1}{2} \mp \frac{\Delta e}{z} \right) \right)$ if $d \le a_v < 2 \cdot d$ the value V_{Ed} is reduced by the coefficient $\beta = \frac{a_v}{2 \cdot d}$	$k_{dg} = \frac{32}{16 + d_g} \ge 0,75;$
prEC 2 [6]	$\tau_{Rd,c} = \frac{V_{Rd,c}}{b_w \cdot d} = \frac{0,6}{\gamma_c} \cdot \left(100 \cdot \rho_l \cdot f_{ck} \cdot \frac{d_{dg}}{d} \right)^{1/3},$ $\tau_{Rd,c} \ge \tau_{Rd,c,min},$ when $\tau_{Rd,c\,min} = \frac{10}{\gamma_c} \cdot \sqrt{\frac{f_{ck}}{f_{yd}} \cdot \frac{d_{dg}}{d}},$ if $d \le a_v < 2 \cdot d$ the value V_{Ed} is reduced by the coefficient $\beta = \frac{a_v}{2 \cdot d}$	if $f_{ck} \le 60 MPa;$ $d_{dg} = 16 + D_{lower} \le 40$ if $f_{ck} > 60 MPa;$ $d_{dg} = 16 + D_{lower} \cdot (60 / f_{ck})^2 \le 40$ $\rho_l = \frac{A_{sl}}{b_w \cdot d};$ if $a_{cs} \le 4d$ $d = a_v = \sqrt{\frac{a_{cs}}{4} \cdot d}$ when $a_{cs} = \lvert M_{Ed} / V_{Ed} \rvert \ge d$

Despite the different methods for obtaining the design equations of the shear resistance models included in the current EC2 and the project prEC2, the latter are quite similar both as recording and in the list of basic variables included in these models. The main difference should be considered that the prEN1992 model attempts to take into account the scale factor (through the ratio d_{dg} / d). At the same time, in prEC2, the value of the coefficient $C_{Rd,c}$ was changed and a different form of notation $\tau_{Rdc,min}$ was proposed.

Some problems associated with estimating the accuracy of the shear resistance models

The shear resistance models included in the actual structural codes are still empirical or semi-empirical. They are based on different types of tests performed under different conditions (in particular, calibrations of empirical coefficients $C_{Rd,c}$).

We should bear in mind that the databases of experimental results used for statistical assessment of the model uncertainties are not always homogeneous and represent the complete sets of input basic variables necessary for performing calculations by theoretical models. For example, at present, extensive databases have been collected containing the results of shear resistance tests of reinforced concrete beams Instance, at present, extensive databases have been collected containing the results of shear tests of the different reinforced concrete beams. However, most of the recent database comprises the results of tests of rectangular beams with the section depth up to 600 mm, tested by concentrated forces applied in the span (only about 8% of test data are beams tested with a uniformly distributed load). To eliminate bending failure mode, most of the beams have, as a rule, so high values of the longitudinal reinforcement ratio ρ_l that they are unrealistic for practice.

Of course, the methodological approaches taken during testing do not fully simulate the physical behavior of an element during shear (for instance, plane stress-strain state).

Another, and even more serious problem relates to the development of empirical models of shear resistance against a background of sets of test results. Moreover, it should be borne in mind that most of the test results from the analyzed databases were obtained on specimens that are not representative of respect to structural elements used in engineering practice, the behavior of which they should model.

As a typical example, we can present the model for estimation of the shear resistance of deep elements without stirrups, included in the current standards EN1992 [1].

Obviously, the proposed model can indeed be most suitable for checking the ultimate limit state of punching of the solid slabs under concentrated (local) load, which, for practical and economic reasons, do not have shear reinforcement (stirrups).

At the same time, actual structural code requirements prohibit reinforced concrete beams without stirrups for practice. In structural elements subjected to bending moments and shear forces, according to the standards [1, 5-6], we have to set the minimum amount of stirrups, even when the condition $V_{Rd,c} \geq V_{Ed}$ is met.

As noted in [10], the sensitivity of slabs to local defects and damages (for example, caverns, unconsolidated places, etc.) is much lower than that of beams. In addition, tests of beams are almost always performed by concentrated forces applied in the immediate vicinity of the support (as a rule, the shear span a / d is from 2.0 to 6.0). With such a test scheme, the maximum shear force coincides with the maximum moment, and, in fact, in the slabs on the supports, the maximum shear force V_{Ed} acts, which decreases to zero in the section with the maximum bending moment M_{Ed} under a uniformly distributed load.

Database containing test results for beam elements without stirrups

We carried the estimation of the uncertainties of the shear models with the usage of test results from our own experimental database, which included 377 beams without stirrups with a wide range of the investigated basic variables. The experimental database was compiled based on the results of laboratory studies, described in detail in the article [8].

The ranges of variation of the main parameters of the analyzed beam elements are presented in Tables 2 and 3.

All beams included in the database (see tables 2 and 3) have a rectangular cross-section, single-span and simply supported, subjected to one or two concentrated forces applied in the span or uniformly distributed load.

Table 2 – Parameters of beam elements subjected to point loading in span

Autor	Number of samples	b, mm	d, mm	ρ_l, %	f_{cm}, MPa	a/d	V_{exp}, kN
Morrow, Viest (1957)	12	305	363 – 375	1,24 – 3,83	14,7 – 45,7	2,76 – 7,86	88,96 – 177,9
Kim, Park (1994)	16	170 – 300	142 – 915	1,01 – 4,68	53,7	3 – 4,5	39,34 – 332,1
Collins, Kuchma (1999)	21	169 – 300	110 – 925	0,5 – 1,03	36 – 99	2,5 – 3,07	40 – 249
Kani, Huggins, Wiltkopp (1979)	32	155	135 – 1097	0,5 – 2,84	17,7 – 34,5	2,5 – 7	24,5 – 165,1
Johnson, Ramirez (1998)	1	305	610	2,49	55,8	3,1	191,3
Elzanaty, Nilson, Slate (1986)	11	177,8	273	1 – 2,5	20,6 – 79,2	4 – 6	44,81 – 78,53
Mphonde, Frantz (1984)	12	152	298	2,32 – 3,36	22,4 – 101,8	2,5 – 3,6	64,6 – 117,9
Islam, Pam, Kwan (1998)	10	150	205	2,02 – 3,22	26,6 – 83,3	2,9 – 3,94	45,5 – 96,9
Ahmad, Khaloo, Poveda (1986)	14	127	184 – 208	1,77 – 6,64	60,8 – 67	2,7 – 4	44,48 – 75,63
Yoon, Cook, Mitchell (1996)	3	375	655	2,8	36 – 87	3,23	249 – 327
Ahmad, Park, El-Dash (1995)	4	102 – 127	178 – 215,9	1,04 – 2,07	40,3 – 89,1	3 – 3,7	19,79 – 43,39
Bazant, Kazemi (1991)	18	38,1	40,6 – 165,1	1,65	46,8	3	2,95 – 10,14
Thorentfeldt, Drangsholt (1990)	16	150 – 300	207 – 442	1,82 – 3,23	54 – 97,7	3 – 4	56,16 – 280,7
Cladera (2002)	4	200	359	2,24	49,9 - 87	3,01	99,69 – 117,9
Adebar, Collins (1996)	6	290 – 360	178 – 278	1 – 3,04	46,2 – 58,9	2,88 – 4,49	74,3 – 128
Xie, Ahmad, Yu, Nino, Chung (1994)	2	127	215,9	2,07	37,7 – 98,9	3	36,68 – 45,72
Salandra, Ahmad (1989)	4	101,6	171,4	1,45	52,1 – 69,1	2,59 – 3,63	20,02 – 29,8
Kulkarni, Shah (1998)	3	102	152	1,37	41,9 – 45	3,5 – 5	19,52 – 24,24
Gonzalez-Fonteboa (2002)	4	200	306	2,87 – 2,93	39,65 – 46,77	3,28	83,88 – 100,5
Hou, Chen, Xu (2015)	3	120	146	3,25	48,85	2,06 – 4,11	29,15 – 94,16
Moody, Viest, Elstner, Hognestad (1954)	21	152 – 178	262 – 533	1,62 – 4,25	17,3 – 36,7	1,52 – 3,41	51,2 – 436,1
Mathey, Watstein (1963)	16	203	403	0,75 – 3,05	21,9 – 27	1,51	180 – 313
Kani (1967)	17	154	132 – 1097	2,58 – 2,84	24,8 – 31,5	1 – 2,5	51,4 – 585,6
Papadakis (1996)	8	140	200	0,8 – 1,2	25	1,5 – 2,5	42,6 – 103,8
Leonhardt, Walther (1961-1962)	8	190	274	2,04	30	1 – 5,83	60,3 – 388,3
Van Den Berg (1962)	30	229	359	4,53	19,1 – 50,3	2,76 – 4,88	99,2 – 177,9
Cao	3	300	1845 – 1925	0,36 – 1,52	27 – 34	2,9	224 – 402
Niwa	3	300 – 600	1000 – 2000	0,14 – 0,28	25,4 – 28	2,98	227 – 804
Quach	1	250	3840	0,66	43,2	3,13	342,3
Sherwood	2	300	1400	0,83	39	2,90	242 – 265

Table 3 – Parameters of beams subjected to uniformly distributed loading

Autor	Number of samples	b, mm	d, mm	ρ_l, %	f_{cm}, MPa	L, mm	V_{exp}, kN
Krefeld, Thurston (1966)	51	152,4 – 254	239,8 – 482,6	1,31 – 4,28	11,2 – 37,2	1829 – 4877	48,7 – 636,5
Shioya (1989)	8	158 – 1500	200 – 3000	0,4	21,2 – 28,5	2161 – 32805	36,1 – 1927,5
Brown, Bayrak (2006)	1	203	406	3,07	26,9	2439	336,7
Stanik, Bentz, Collins (2007)	3	113 – 300	230 – 617	0,76 – 1,15	31,3 – 35,8	1007 – 5815	64,1 – 255,5
Smith (1970)	3	150	200	2,01	28 – 36,2	2452 – 3664	50,5 – 59
de Cossio, Seiss (1960)	6	152	252 – 276	1,01 – 1,35	19,2 – 41,2	1674 – 2795	59,9 – 135

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Estimation methodology and results of estimation

We carried the estimation of the shear resistance models from Table 1 based on random samples made up of the ratios V_{theo} / V_{test}. Considering the fact that the shear strength depends *mainly* on the variation in the concrete compressive strength $\sqrt{f_{ck}}$, we checked the requirements declared by the researches, according to which the 5% - quantile of the distribution of the ratio V_{theo} / V_{test} should be close to 1. At the same time, at the first stage, a suitable probability distribution function was established for the samples N = 35 of results using the Kolmogorov – Smirnov test, and then for the selected distribution the value of the 5% -quantile of the statistical distribution of the ratio V_{theo} / V_{test} was calculated. Additionally, the 5% quantile of the statistical distribution was calculated using the method of Order statistics detailed in [2, 4]. The method of Order (non-parametric) statistics allows calculating the quantile of a given order without determining the probability density distribution function (*pdf*) and for a required confidence level (γ = 0,5; 0,75; 0,9).

The results of estimations of 5 % quantiles using both the empirical distributions and the method based on Order statistics theory are shown in Table 4.

Table 4 – Calculation results of the 5% -quantile of the distribution of the shear resistance ratio V_{theo} / V_{test} according to various models

Prediction model	The value of the 5% -quantile of the distribution of the V_{theo} / V_{exp} ratio				
	Empirical distribution		Method based on Order statistics for confidence level		
	pdf	Value	γ=0,5	γ=0,75	γ=0,9
Beam elements subjected to uniformly distributed loading at L/d < 10,0					
Model Code 2010 (LoA I)	G	0,146	0,143	0,135	0,122
Model Code 2010 (LoA II)	LN	0,338	0,325	0,307	0,276
prEN 1992-1-1	N	0,353	0,354	0,334	0,301
EN 1992-1-1	N	0,208	0,205	0,194	0,174
Beam elements subjected to uniformly distributed loading at L/d ≥ 10,0					
Model Code 2010 (LoA I)	LN	0,248	0,299	0,283	0,256
Model Code 2010 (LoA II)	G	0,659	0,701	0,673	0,625
prEN 1992-1-1	LN	0,824	0,838	0,834	0,826
EN 1992-1-1	G	0,501	0,563	0,546	0,517
Beam elements subjected to point loading at a/d < 2,0					
Model Code 2010 (LoA I)	G	0,144	0,144	0,142	0,139
Model Code 2010 (LoA II)	G	0,350	0,339	0,334	0,325
prEN 1992-1-1	N	0,375	0,381	0,371	0,353
EN 1992-1-1	N	0,307	0,303	0,299	0,293
Beam elements subjected to point loading at a/d ≥ 2,0					
Model Code 2010 (LoA I)	LN	0,323	0,381	0,313	0,255
Model Code 2010 (LoA II)	N	0,560	0,673	0,650	0,639
prEN 1992-1-1	N	0,628	0,749	0,689	0,622
EN 1992-1-1	LN	0,614	0,711	0,686	0,653
Note: LN – lognormal distribution; N – normal distribution; G – Gumbel distribution.					

As seen from the results shown in Table 4 for various cases of loading, including slender and rigid beams subjected to uniformly distributed loading, practically none of the analyzed models gives the expected value of the ratio $V_{theo} / V_{test} \approx 1{,}0$ in the 5 % -quantile, which was declared, for example, in [9]. The closest to unity values of the ratio V_{theo} / V_{test} are given by the prEC2 design model for the slender beams ($L / d \geq 10$) subjected to uniformly distributed loading (0.824 - with an empirical N-distribution and 0.826 with an estimate by the method of order (non-parametric) statistics with confidence level γ = 0,90). If we rely on the obtained results, we can conclude that almost all the analyzed models provide quite significant reserves (in particular, for the beams with shear span to depth ratio $a / d < 2{,}0$ and rigid beams with $L / d < 10$ analyzed models underestimate the shear resistance by up to 7 times!). The following question arises: how it can be explained? Is the result obtained random or are the empirical coefficients in the models specially selected in this way? These questions require additional analysis, considering the previously shown errors associated with the estimation, starting with the formation of reliable samples of experimental data.

However, we can make some preliminary remarks. So, according to prEC2, the shear resistance model has the following formulation:

$$\tau_{Rd,c} = \frac{0{,}6}{\gamma_c} \cdot \left(100 \cdot \rho_l \cdot f_{ck} \cdot \frac{d_{dg}}{d} \right)^{1/3} \geq \tau_{Rdc,min}, \quad (1)$$

The coefficient (partial factor for concrete) is used to transform from the characteristic value of the shear resistance $\tau_{Rk,c} = f(f_{ck})$ to its design value $\tau_{Rd,c}$ .It should be noted that when equation (1) was derived, the authors of [9] obtained a coefficient equal to 0.87. If we assume that the transition to the design value of shear resistance $\tau_{Rd,c}$ is equivalent to the application of the design value of concrete compressive strength (f_{ck} / γ_c) in the design model (1), then the characteristic value of the shear resistance should correspond to a 5% quantile of the resistance distribution.

The estimation of the reliability of the design shear resistance models was carried out on the basis of samples of experimental data that have the same or very close parameters with variable values f_{cm} . Next, a sample of experimental data with close values f_{cm} is estimated (selected experimental values of shear stress in Figure 1). We applied the method based on the order (non-parametric) statistics for assessing the shear stress value corresponding to the 5% quantile shear resistance distribution with a required confidence level. The experimental values of the shear stress in 5% quantiles are compared with the dependence function $\tau_{Rc,theo} = f(f_{ck})$ of the estimated design model (see Figure 1). We may consider the shear resistance model conditionally accurate with an assigned confidence level if the ratio $\tau_{Rc,theo} / \tau_{Rc,exp,5\%-кв.} \approx 1{,}0$. Otherwise, the model is adjusted by changing the value of the coefficient until the model is suitable for the accepted criterion.

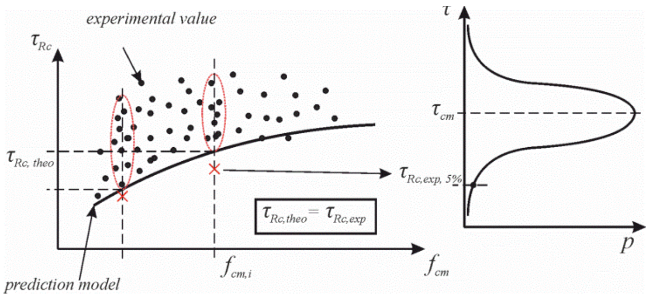

Figure 1 – Estimation of the shear resistance model uncertainty

Some problems of this method are in the difficulty of selecting experimental data with the same or close parameters and variable values f_{cm} . Figures 2-4 show the diagrams of the estimation of the shear resistance models according to the EC2, prEC2 and *fib* Model Code 2010 (LoA II), for beams subjected to point and uniformly distributed loadings.

The results of estimating the reliability of shear resistance models according to the described method based on non-parametric statistics for various types of loading are presented in Tables 5-13.

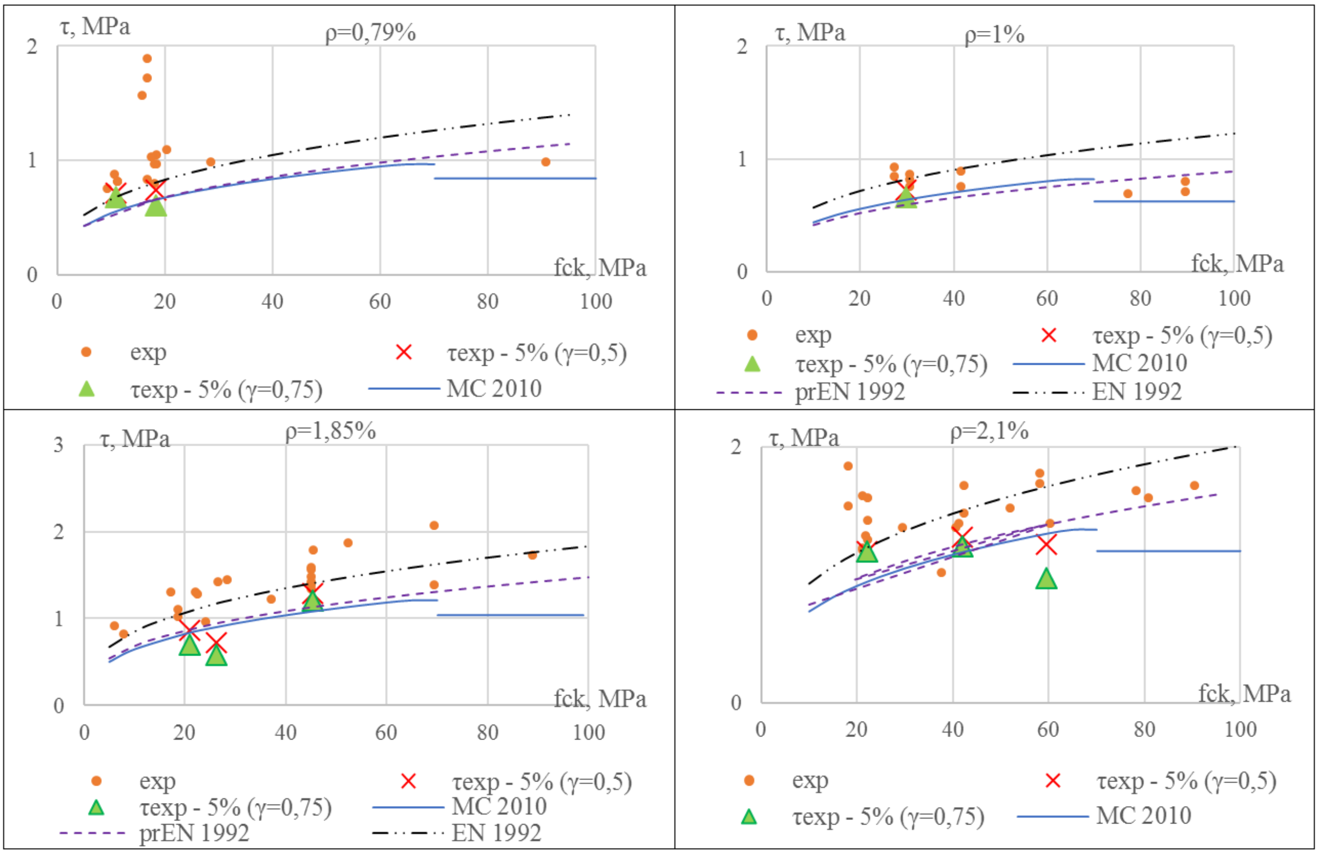

Figure 2 – Estimates of various shear resistance models at $a / d \geq 2{,}0$

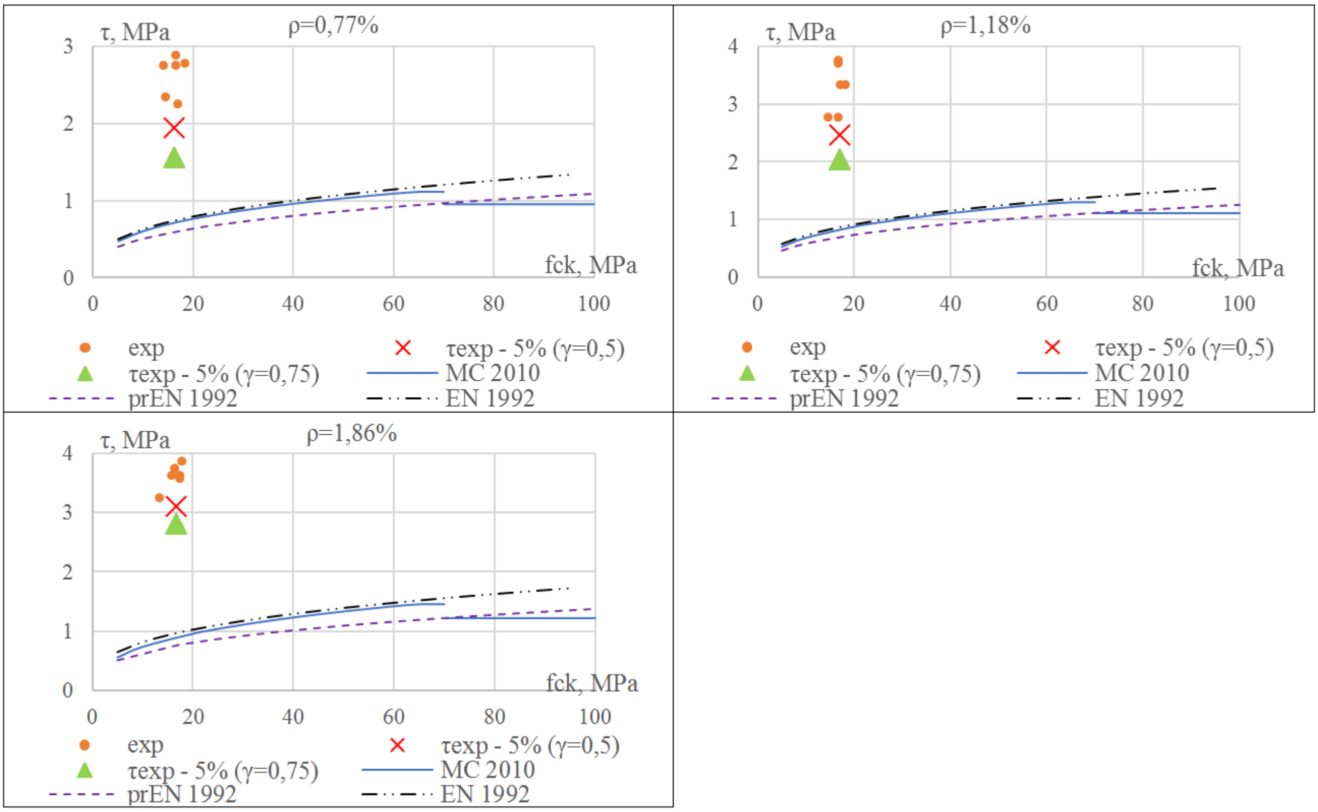

Figure 3 – Estimates of various shear resistance models at $a / d < 2{,}0$

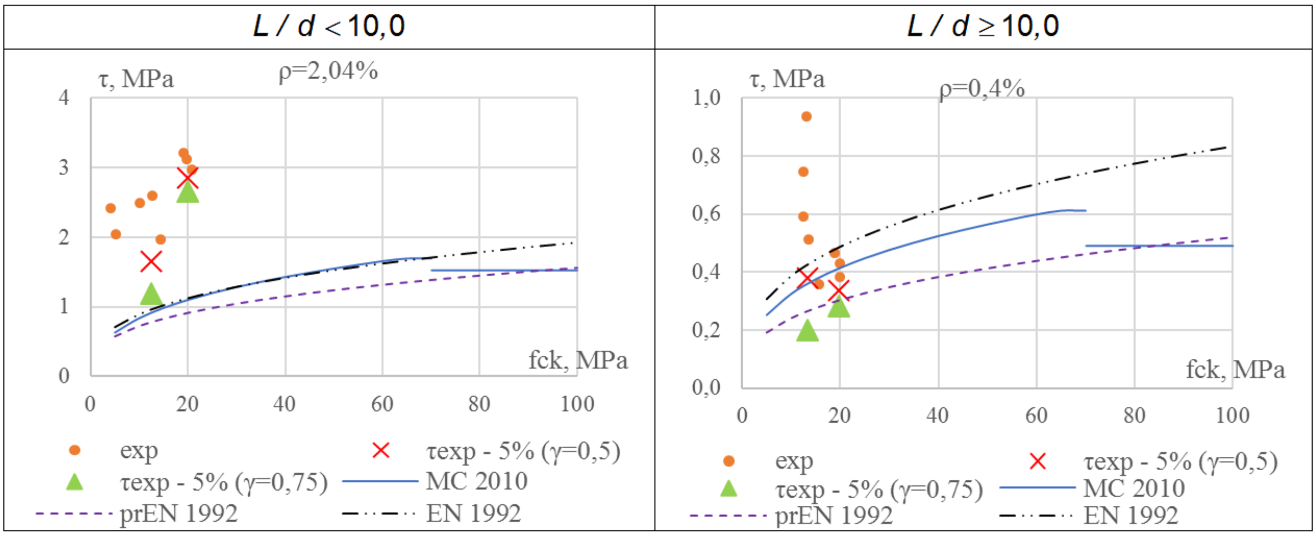

Figure 4 – Estimates of various shear resistance models (uniformly distributed loading)

Table 5 – The results of estimating the reliability of the shear resistance model according to the EC2 ($a/d \geq 2{,}0$)

Pre-diction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio τ_{theo}/τ_{test} for 5% -quatile τ_{test}
EC2	$\rho_l=0{,}79\%$	11,1	$\gamma=0{,}5$	0,71	0,68	0,96
			$\gamma=0{,}75$	0,66		1,04
		18,3	$\gamma=0{,}5$	0,73	0,81	1,12
			$\gamma=0{,}75$	0,61		1,34
	$\rho_l=1{,}0\%$	29,8	$\gamma=0{,}5$	0,71	0,59	0,84
			$\gamma=0{,}75$	0,65		0,92
	$\rho_l=1{,}85\%$	21	$\gamma=0{,}5$	0,86	1,09	1,26
			$\gamma=0{,}75$	0,67		1,62
		26,7	$\gamma=0{,}5$	0,70	1,18	1,69
			$\gamma=0{,}75$	0,57		2,09
		45,7	$\gamma=0{,}5$	1,27	1,41	1,11
			$\gamma=0{,}75$	1,20		1,17
	$\rho_l=2{,}1\%$	22,4	$\gamma=0{,}5$	1,17	1,22	1,04
			$\gamma=0{,}75$	1,16		1,05
		42,2	$\gamma=0{,}5$	1,29	1,51	1,17
			$\gamma=0{,}75$	1,21		1,24
		59,6	$\gamma=0{,}5$	1,23	1,69	1,38
			$\gamma=0{,}75$	0,97		1,74

Table 6 – The results of estimating the reliability of the shear resistance model according to the EC2 ($a/d<2{,}0$)

Pre-diction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio τ_{theo}/τ_{test} for 5% -quatile τ_{test}
EC2	$\rho_l=0{,}77\%$	16,5	$\gamma=0{,}5$	1,94	0,75	0,38
			$\gamma=0{,}75$	1,54		0,48
	$\rho_l=1{,}18\%$	17,3	$\gamma=0{,}5$	2,43	0,87	0,36
			$\gamma=0{,}75$	2,02		0,43
	$\rho_l=1{,}86\%$	16,8	$\gamma=0{,}5$	3,09	0,97	0,31
			$\gamma=0{,}75$	2,81		0,34

Table 7 – The results of estimating the reliability of the shear resistance model according to the EC2 (uniformly distributed loading)

Pre-diction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio τ_{theo}/τ_{test} for 5% -quatile τ_{test}
$L/d<10{,}0$						
EC2	$\rho_l=2{,}04\%$	12,7	$\gamma=0{,}5$	1,63	0,97	0,59
			$\gamma=0{,}75$	1,19		0,81
		20,3	$\gamma=0{,}5$	2,82	1,13	0,40
			$\gamma=0{,}75$	2,66		0,43
$L/d\geq 10{,}0$						
EC2	$\rho_l=0{,}4\%$	13,5	$\gamma=0{,}5$	0,37	0,43	1,15
			$\gamma=0{,}75$	0,20		2,14
		20	$\gamma=0{,}5$	0,33	0,49	1,48
			$\gamma=0{,}75$	0,27		1,78

Table 8 – The results of estimating the reliability of the shear resistance model presented in the prEC2 ($a / d \geq 2{,}0$)

Pre-diction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio $\tau_{theo} / \tau_{test}$ for 5% -quatile τ_{test}
prEC2	$\rho_l = 0{,}79\%$	11,1	$\gamma = 0{,}5$	0,71	0,56	0,79
			$\gamma = 0{,}75$	0,66		0,85
		18,3	$\gamma = 0{,}5$	0,73	0,66	0,90
			$\gamma = 0{,}75$	0,61		1,08
	$\rho_l = 1{,}0\%$	29,8	$\gamma = 0{,}5$	0,71	0,59	0,83
			$\gamma = 0{,}75$	0,65		0,91
	$\rho_l = 1{,}85\%$	21	$\gamma = 0{,}5$	0,86	0,88	1,02
			$\gamma = 0{,}75$	0,67		1,31
		26,7	$\gamma = 0{,}5$	0,70	0,95	1,36
			$\gamma = 0{,}75$	0,57		1,67
		45,7	$\gamma = 0{,}5$	1,27	1,14	0,90
			$\gamma = 0{,}75$	1,20		0,95
	$\rho_l = 2{,}1\%$	22,4	$\gamma = 0{,}5$	1,17	1,01	0,86
			$\gamma = 0{,}75$	1,16		0,87
		42,2	$\gamma = 0{,}5$	1,29	1,24	0,96
			$\gamma = 0{,}75$	1,21		1,03
		59,6	$\gamma = 0{,}5$	1,23	1,39	1,13
			$\gamma = 0{,}75$	0,97		1,43

Table 9 – The results of estimating the reliability of the shear resistance model presented in the prEC2 ($a / d < 2{,}0$)

Pre-diction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio $\tau_{theo} / \tau_{test}$ for 5% -quatile τ_{test}
prEC2	$\rho_l = 0{,}77\%$	16,5	$\gamma = 0{,}5$	1,94	0,60	0,31
			$\gamma = 0{,}75$	1,54		0,39
	$\rho_l = 1{,}18\%$	17,3	$\gamma = 0{,}5$	2,43	0,70	0,29
			$\gamma = 0{,}75$	2,02		0,35
	$\rho_l = 1{,}86\%$	16,8	$\gamma = 0{,}5$	3,09	0,76	0,25
			$\gamma = 0{,}75$	2,81		0,27

Table 10 – The results of estimating the reliability of the shear resistance model presented in the prEC2 (uniformly distributed loading)

Pre-diction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio $\tau_{theo} / \tau_{test}$ for 5% -quatile τ_{test}
$L / d < 10{,}0$						
prEC2	$\rho_l = 2{,}04\%$	12,7	$\gamma = 0{,}5$	1,63	0,78	0,48
			$\gamma = 0{,}75$	1,19		0,66
		20,3	$\gamma = 0{,}5$	2,82	0,92	0,33
			$\gamma = 0{,}75$	2,66		0,35
$L / d \geq 10{,}0$						
prEC2	$\rho_l = 0{,}4\%$	13,5	$\gamma = 0{,}5$	0,37	0,27	0,73
			$\gamma = 0{,}75$	0,20		1,35
		20	$\gamma = 0{,}5$	0,33	0,30	0,91
			$\gamma = 0{,}75$	0,27		1,11

Table 11 – The results of estimating the reliability of the shear resistance model according to the *fib* Model Code 2010 (LoA II) ($a / d \geq 2{,}0$)

Predic-tion model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quan-tile τ_{test}, MPa	τ_{theo}, MPa	Ratio $\tau_{theo} / \tau_{test}$ for 5% -quatile τ_{test}
fib MC 2010 (LoA II)	$\rho_l = 0{,}79\%$	11,1	$\gamma = 0{,}5$	0,71	0,56	0,79
			$\gamma = 0{,}75$	0,66		0,85
		18,3	$\gamma = 0{,}5$	0,73	0,66	0,90
			$\gamma = 0{,}75$	0,61		1,08
	$\rho_l = 1{,}0\%$	29,8	$\gamma = 0{,}5$	0,71	0,64	0,90
			$\gamma = 0{,}75$	0,65		0,99
	$\rho_l = 1{,}85\%$	21	$\gamma = 0{,}5$	0,86	0,84	0,98
			$\gamma = 0{,}75$	0,67		1,25
		26,7	$\gamma = 0{,}5$	0,70	0,91	1,30
			$\gamma = 0{,}75$	0,57		1,60
		45,7	$\gamma = 0{,}5$	1,27	1,09	0,86
			$\gamma = 0{,}75$	1,20		0,91
	$\rho_l = 2{,}1\%$	22,4	$\gamma = 0{,}5$	1,17	0,95	0,81
			$\gamma = 0{,}75$	1,16		0,82
		42,2	$\gamma = 0{,}5$	1,29	1,18	0,92
			$\gamma = 0{,}75$	1,21		0,98
		59,6	$\gamma = 0{,}5$	1,23	1,32	1,07
			$\gamma = 0{,}75$	0,97		1,36

Table 12 – The results of estimating the reliability of the shear resistance model presented in the *fib* Model Code 2010 (LoA II) ($a / d < 2{,}0$)

Prediction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio $\tau_{theo} / \tau_{test}$ for 5% -quatile τ_{test}
fib MC 2010 (LoA II)	$\rho_l = 0{,}77\%$	16,5	$\gamma = 0{,}5$	1,94	0,72	0,37
			$\gamma = 0{,}75$	1,54		0,47
	$\rho_l = 1{,}18\%$	17,3	$\gamma = 0{,}5$	2,43	0,83	0,34
			$\gamma = 0{,}75$	2,02		0,41
	$\rho_l = 1{,}86\%$	16,8	$\gamma = 0{,}5$	3,09	0,89	0,29
			$\gamma = 0{,}75$	2,81		0,32

Table 13 – The results of estimating the reliability of the shear resistance model according to the *fib* Model Code 2010 (LoA II) (uniformly distributed loading)

Prediction model	Reinforcement ratio ρ_l	f_{ck}, MPa	Confidence level, γ	5% - quantile τ_{test}, MPa	τ_{theo}, MPa	Ratio $\tau_{theo} / \tau_{test}$ for 5% -quatile τ_{test}
$L / d < 10{,}0$						
fib MC 2010 (LoA II)	$\rho_l = 2{,}04\%$	12,7	$\gamma = 0{,}5$	1,63	0,92	0,56
			$\gamma = 0{,}75$	1,19		0,77
		20,3	$\gamma = 0{,}5$	2,82	1,11	0,39
			$\gamma = 0{,}75$	2,66		0,42
$L / d \geq 10{,}0$						
fib MC 2010 (LoA II)	$\rho_l = 0{,}4\%$	13,5	$\gamma = 0{,}5$	0,37	0,36	0,97
			$\gamma = 0{,}75$	0,20		1,8
		20	$\gamma = 0{,}5$	0,33	0,42	1,27
			$\gamma = 0{,}75$	0,27		1,56

Conclusions

Based on the results of evaluating the reliability of the calculated shear resistance models presented in this work, the following conclusions can be drawn:

1. One of the key characteristics affecting the accuracy of estimating the reliability of prediction models is the need for a reasonable choice of the probability distribution function based on the empirical sample obtained. Due to the asymmetric distribution, difficulties arise in calculating the 5% quantile.
2. Since in most prediction models the conversion from the characteristic value of concrete strength at shear $\sqrt{f_{ck}}$ is performed by dividing by a partial coefficient γ_c = 1,5, it would be methodologically correct for the ratio of theoretical and experimental resistance $V_{theo} / V_{test} \cong 1$ to correspond to the 5% quantile of the distribution, and not to the average value.
3. Taking into account the above remarks, a proprietary method for estimating the reliability of shear resistance models was proposed, based on the method of ordinal statistics, which does not require the determination of the probability and density distribution function, and also allows calculating the quantile of the required order for a predetermined security.

The results of estimating the reliability of the models according to the generally accepted and proposed methods show that practically none of the analyzed models gives the expected ratio $V_{theo} / V_{test} \cong 1{,}0$ in the 5% quantile. The closest to unity values of the ratio V_{theo} / V_{test} are given by the considered design models for flexible beams subjected to uniformly distributed loading ($L / d \geq 10$) and beams with shear span to depth ratio $a / d \geq 2{,}0$, subjected to point loading. For rigid beams ($a / d < 2{,}0$ and $L / d < 10$), all the models under study provide a fairly significant margin. Based on the results obtained, the question arises about the applicability of these models to the required level of reliability.

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