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RAIN SURFACE RUNOFF MODELING USING CELLULAR AUTOMATON

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Abstract

The numerical solution of a system of two-dimensional shallow water (Saint-Venant) differential equations in partial derivatives for modeling the dynamics of surface runoff using digital elevation models obtained by laser altimetry leads to a significant increase in the computational load on the computer.

The article offers a view on reasonable simplifications for the shallow water model, the two-dimensional cellular automaton for the modeling of rain surface runoff is proposed along with adequacy analysis of the calculations performed in their practical application.

Keywords: shallow water equations, rain surface runoff, cellular automaton.

МОДЕЛИРОВАНИЕ ДОЖДЕВОГО СТОКА ПРИ ПОМОЩИ КЛЕТОЧНОГО АВТОМАТА

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Реферат

Выполнение численного решения системы двумерных дифференциальных уравнений мелкой воды (Сен-Венана) в частных производных для моделирования динамики движения водных потоков по цифровым моделям рельефа, построенным на основе лазерного сканирования земной поверхности с высокой разрешающей способностью, приводит значительному повышению вычислительной нагрузки на ЭВМ.

В данной статье рассматриваются способы обоснованного упрощения модели движения воды по земной поверхности, предлагается один из вариантов применения клеточного автомата для моделирования дождевого стока и оценка адекватности выполняемых расчетов при их практическом применении.

Ключевые слова: уравнения мелкой воды, дождевой сток, клеточный автомат.

Introduction

Modeling of rainfall runoff is important as for predicting the regime of water supply to rivers, lakes, and reservoirs, so for preventing floods in urbanized areas. The significant improvement in the quality of 3D terrain models due to the development of remote sensing of the Earth's surface has led to increased requirements for the performance of hydraulic modeling software. The aim of the presented work is to review reasonable ways to simplify the model of water flow using digital models and to consider the features of using cellular automaton for such purposes with an analysis of the adequacy of the calculations performed.

Physically based hydraulic approach and ways to simplify it

The theoretical basis for rain surface runoff modeling is a physically substantiated calculation of water flows dynamics, which is described by a system of two-dimensional equations of shallow water (Saint-Venant) [1] having the following form[2, 3, 18]:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = S \qquad (1)$$

$$U = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix};\ F = \begin{pmatrix} hu \\ hu^2 + gh^2/2 \\ huv \end{pmatrix};\ G = \begin{pmatrix} hv \\ huv \\ hv^2 + gh^2/2 \end{pmatrix};\ S = \qquad (2)-(5)$$

$$S_{fx} = \frac{-n^2 u\sqrt{u^2+v^2}}{h^{4/3}};\ S_{fy} = \frac{-n^2 v\sqrt{u^2+v^2}}{h^{4/3}} \qquad (6)$$

where t is time; x and y are horizontal coordinates; h is a water depth; u and v are depth-averaged projections of the water velocity vector onto the horizontal coordinate axes X and Y; g is acceleration of gravity; S_{0x} and S_{0y} are bottom slopes in the direction of the horizontal coordinate axes; S_{fx} and S_{fy} are friction slopes in the direction of the horizontal coordinate axes [2, 4, 18]; n is the Manning roughness coefficient; r is the rainfall intensity; f are water infiltration losses. If we neglect the inertial components in the equations conservation of momentum, then it is possible to obtain a diffusion approximation to describe the motion of water:

$$\frac{\partial U_d}{\partial t} + \frac{\partial F_d}{\partial x} + \frac{\partial G_d}{\partial y} = S_d \qquad (7)$$

$$U_d = \begin{pmatrix} h \\ 0 \\ 0 \end{pmatrix};\ F_d = \begin{pmatrix} hu \\ gh^2/2 \\ 0 \end{pmatrix};\ G_d = \begin{pmatrix} hv \\ 0 \\ gh^2/2 \end{pmatrix};\ S_d = \qquad (8)-(11)$$

Further simplification of the diffusion equations leads to a kinematic approximation of the description of the water motion:

$$\frac{\partial U_k}{\partial t} + \frac{\partial F_k}{\partial x} + \frac{\partial G_k}{\partial y} = S_k \qquad (12)$$

$$U_k = \begin{pmatrix} h \\ 0 \\ 0 \end{pmatrix};\ F_k = \begin{pmatrix} hu \\ 0 \\ 0 \end{pmatrix};\ G_k = \begin{pmatrix} hv \\ 0 \\ 0 \end{pmatrix};\ S_k = \qquad (13)-(16)$$

Since the river and surface runoff in conditions of a predominantly flat terrain (which, in particular, is typical for the Republic of Belarus) is characterized by a laminar flow, the use of the diffusion approximation

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for describing the movement of water in such conditions is permissible [18, 20].

Application of the cellular automaton to simulate water movement

The above equations can be solved numerically using the finite volume method and the implicit Euler difference scheme [5]. The finite volume method assumes covering the area of computations with a network of small contiguous polygonal cells and allows the researcher to decompose the solution of a two-dimensional problem into a set of interconnected one-dimensional problems to calculate the fluid flow through each side of discrete cells [3, 6].

For a wide class of fluid dynamics problems, including modeling of surface runoff, it is considered acceptable to replace the use of difference schemes for solving systems of partial differential equations with the use of a cellular automaton [7, 9 – 17]. The behavior of a cellular automaton (CA) is completely defined in terms of local dependencies as well as in a wide class of continuous dynamic systems defined by partial differential equations [8]. CA is a complex discrete dynamic system consisting of a finite number of elements called cells, the state of which changes synchronously over time according to a given set of rules [7, 8]. When using a digital elevation model in the form of a set of elevation marks located at the nodes of a regular rectangular mesh to simulate water runoff over the earth's surface, the mesh nodes are interpreted as the centers of independent discrete horizontal areas with a given height value, and the areas themselves (the bases of finite volumes) coincide with the cells of the cellular automaton.

Development of a cellular automaton for modeling rain runoff

The structure of the cellular automaton

Based on [17], a two-dimensional cellular automaton was created for simulating rain runoff, which is a set of $k_i \in K$ elements. The $k_i \in K$ placement coincides with the location of the square-shaped elements of the digital elevation model in the area on which liquid precipitation falls in the form of rain, while it is assumed that the accumulated water does not leave the DEM. Let us denote the set of elements of the cellular automaton located in the J. von Neumann neighborhood with respect to $k_i \in K$ as $k_j \in N^{k_i}$. The subset of $k_j \in N^{k_i}$ elements that can receive a non-zero volume of water by overflow from k_i will be denoted as $k_j \in N^{k_i}_{V_{ij}}$. The following attributes are defined for each k_i element of the cellular automaton (see Figure 1): z_i, Δx, A, n_i is the height (m), the length of the side (m), the area (m^2), and the Manning roughness coefficient of the appropriate DEM element, respectively; d_i is the depth of the water layer (m); $S_{ij} = (wl_i - wl_j)/\Delta x = \Delta wl_{ij}/\Delta x$ is the hydraulic slope between k_i and $k_j \in N^{k_i}$, where $wl_i = d_i + z_i$ and $wl_j = d_j + z_j$ are representing water height (m); $wl_{ij^{max}}$ is the maximum height of the water level among $k_j \in N^{k_i}$; $d_{ij} = wl_i - z_j$ is the effective water depth (m) for the k_i element in direction of $k_j \in N^{k_i}_{V_{ij}}$; $v_{ij} = [(d_{ij})^{2/3}\sqrt{S_{ij}}]\,/n_i$ is the speed of water stream (m/s) between k_i and $k_j \in N^{k_i}_{V_{ij}}$; $Q_{ij} = \Delta x \cdot d_{ij} \cdot v_{ij}$ is water discharge (m^3/s) between k_i and $k_j \in N^{k_i}_{V_{ij}}$; $Q_{ij^{max}}$ is water discharge (m^3/s) between k_i and the neighboring element with the maximum water height among $k_j \in N^{k_i}_{V_{ij}}$; $\Delta t_i = [(wl_i - wl_{ij^{max}}) \cdot A]\,/\,\square$ is the time period (s), during which the height of the water level in the k_i element will be equalized with the neighbor for which the height of the water level was previously determined equal to $wl_{ij^{max}}$ in the process of water flowing from the k_i element to all neighbors for which the $wl_i - wl_j \geq \delta$ condition is valid; $V^{out}_{ij} = Q_{ij} \cdot \Delta T$ is the volume of water transferred from k_i towards the direction of $k_j \in N^{k_i}_{V_{ij}}$, where ΔT is the step of time (s), defined for the cellular automaton at the current simulation iteration; $V^{out}_i = \sum V^{out}_{ij}$ is the total water volume (m^3), which leaves the k_i element in the direction of all $k_j \in N^{k_i}_{V_{ij}}$ at the current simulation iteration; $V_i = \sum V^{out}_{ji}$ is the total water volume (m^3), sourced from all $k_j \in N^{k_i}$ elements in the direction of k_i at the current simulation iteration.

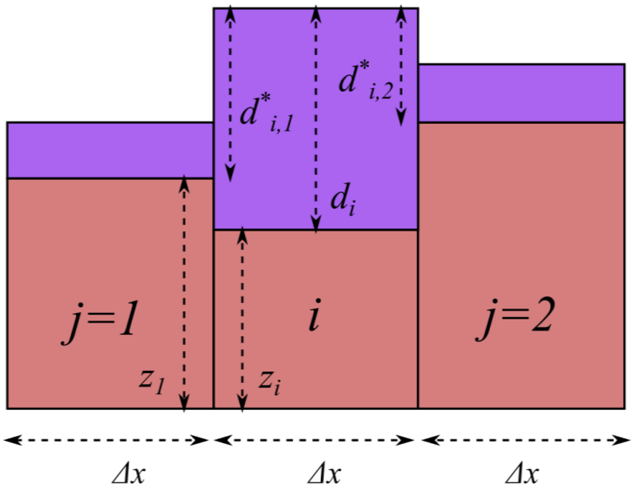

Figure 1 – The value of the effective water depth d_{ij} for the k_i element in the direction of the neighbors $k_j \in N^{k_i}_{V_{ij}}$.

The rule for changing the state of a cellular automaton

Before the start of the runoff simulation iterations, the water depth d_i for each $k_i \in K$ element it is set equal to the thickness of the water layer precipitated in 1 second. The rule of synchronous change in the state of $k_i \in K$ elements of the cellular automaton at each iteration of the simulation is described by the following sequence of steps:

1 The V^{out}_{ij} and V^{out}_i values are zeroed, Δt_i is set equal to ∞, $N^{k_i}_{V_{ij}} \equiv \emptyset$;
2 If the $d_i < \delta(\delta = 0{,}5 \cdot 10^{-3} m)$ condition is true, go to step 8;
3 Calculate $S_{ij} = (wl_i - wl_j)/\Delta x = \Delta wl_{ij}/\Delta x$ for $k_j \in N^{k_i}$;
4 The $k_j \in N^{k_i}$ elements, for which the $S_{ij} > \delta$ condition is true, are included into the $N^{k_i}_{V_{ij}}$ set;
5 Calculate $\Delta t_i = [(wl_i - wl_{ij^{max}}) \cdot A]\,/\,\square$;
6 Accept the $\Delta T = min(\Delta t_i)$ value – find minimal Δt_i among all set of $k_i \in K$;
7 Calculate $V^{out}_{ij} = Q_{ij} \cdot \Delta T$ and $V^{out}_i = \sum V^{out}_{ij}$;
8 Calculate $V_i = \sum V^{out}_{ji}$;
9 Decrease d_i with the V^{out}_i/A value;
10 Add the total of V_i/A and the thickness of precipitated water layer, which precipitated during the ΔT time interval, to d_i.

Analysis of the adequacy of the proposed model

The analysis of the adequacy of the proposed method for modeling rainfall runoff was carried out based on the execution of the scenario of rainfall in an urbanized area (see Figures 2 and 3) given in [19, p. 99] under the title "Test 8A: Rainfall and point source flow in urban areas". According to the scenario, liquid precipitation occurs on an urban terrain with an area of 0.388 km^2 with an intensity of 400 mm/h during 3 minutes, then, 16 minutes after the end of the rain from a point source, water begins to flow for 35 minutes with a peak water discharge of 5 m^3/s, and after 17 minutes no water runoff from the modeling site beyond its boundaries occurs. The horizontal resolution of the provided digital elevation model is 2 m, the Manning roughness coefficient of the DEM elements representing paved areas of the territory (roads and sidewalks) is 0.02, and the roughness of the rest of the area is 0.05.

Figure 2 – General view of a section of an urbanized territory used in the scenario of modeling rain runoff (there are no buildings on the digital elevation model)

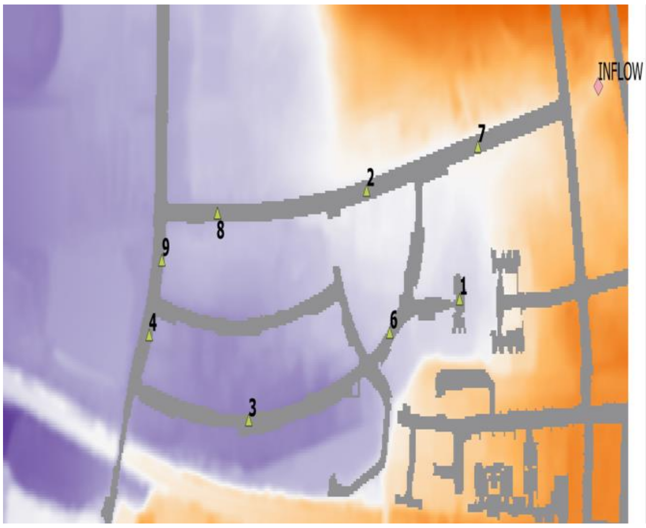

Figure 3 – Image of a digital elevation model for a scenario of rainfall runoff modeling: the control points for measuring the height of the water level are indicated by numbers, the mark "INFLOW" is the location of a point source of water inflow [19, p. 99]

With the help of the software implementation of the proposed cellular automaton, a simulation of rain runoff with a duration of 100 minutes was carried out according to the scenario described in [19]. The results of measuring the height of the water level at control points No. 1 and No. 2 are shown in Figures 4 and 5. For comparison, the figures show the graphs of changes in the water depth at these points, obtained by the authors of the test scenario using foreign software for hydraulic modeling, based both on the solution two-dimensional shallow water equations without simplifications, and on the solution of simplified diffusion equations (in this case, the RFSM EDA software package was used [20]).

The Pearson correlation coefficient was used as a quantitative characteristic of the differences between the results of the cellular automaton operation and the reference software. The correlation of the obtained values of the water level height with the results of solving the shallow water equations was 0.873 and 0.782 for control points No. 1 and No. 2, respectively, and the correlation with the solution of simplified diffusion equations was 0.943 and 0.913 for the same control points, respectively.

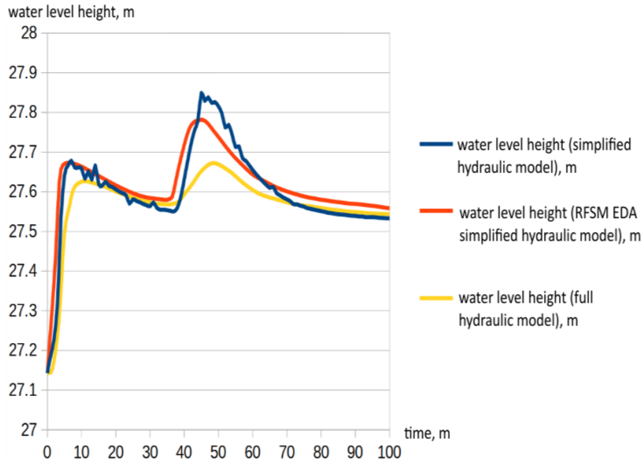

Figure 4 – Simulation results of the water level height change at the control point No. 1

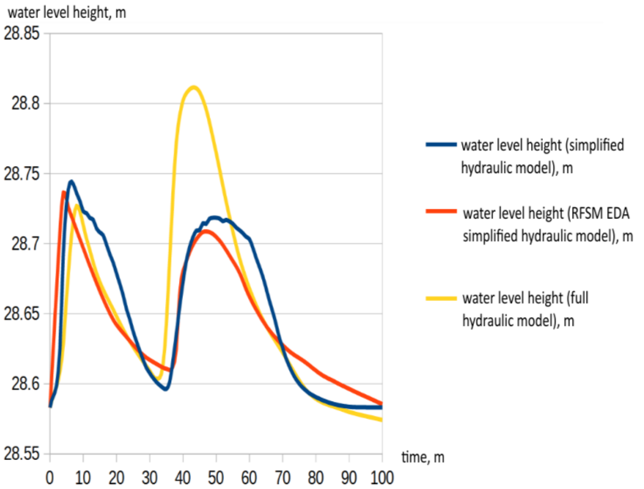

Figure 5 – Simulation results of the water level height change at the control point No.2

Conclusion

Considering the correspondence between the results of the analysis of water movement using a high-resolution digital elevation model using mathematical methods with varying degrees of simplification of the ongoing physical processes, it is possible to note noticeable differences when calculating the dynamics of transient processes between the complete and simplified hydraulic models. Despite these differences, a simplified hydraulic model implemented using a cellular automaton makes it possible to adequately assess not only the final picture of water spreading over the terrain [21, 22, 23], but also the time of the end of the process of water movement, which can be important when forecasting floods in an urbanized area.

References

1. Kivva, S. L. Two-dimensional modeling of rainfall and sediment transport in small catchments / S. L. Kivva, M. I. Zheleznyak // Applied Hydromechanics. – 2002. – T. 4 (76), № 1. – P. 34–43.
2. Nikishov, V. I. From open flow hydraulics to hydromechanics of river systems / V. I. Nikishov // Applied hydromechanics. – 2007. – T. 9, № 2–3. – P. 103–121.
3. Costabile, P. Comparative analysis of overland flow models using finite volume schemes / P. Costabile, C. Costanzo, F. Macchione // Journal of Hydroinformatics. – 2012. – Vol. 14, № 1. – P. 122–135.
4. Karepova, E. D. Simulation of unsteady water movement in the lower bank of the Boguchanskaya HPP / E. D. Karepova, G. A. Fedorov // Computational technologies. – 2008. – T. 13, № 2. – P. 28–38.

doi.org/10.36773/1818-1112-2021-126-3-88-91

5. Baranov A. V. Hydrological module in the software package "Nymph" / A. V. Baranov [et al.] // Supercomputing and mathematical modeling. Proceedings of the XVII International Conference, Sarov, October 15–19, 2018 / Ed. R. M. Shagalieva / FSUE RFYATs-VNIIEF. – Sarov : FSUE RFYATs-VNIIEF, 2019. – P. 33–37.
6. Smirnov, E. M. The finite volume method as applied to problems of hydro-gas-dynamics and heat transfer in areas of complex geometry / E. M. Smirnov, D. K. Zaitsev // Scientific and technical statements of St. Petersburg State Technical University. – 2004. – T. 2, № 36. – P. 70–81.
7. Dottori, F. A 2d flood inundation model based on cellular automata approach / F. Dottori, E. Todini // XVIII International Conference on Water Resources, Barcelona, Spain, June 21–24, 2010 / International Centre for Numerical Methods in Engineering. – International Centre for Numerical Methods in Engineering, 2010.
8. Toffoli T. Cellular automata machines: trans. with. English / T. Toffoli, M. Margolus. – M .: Mir, 1991 . – 280 p.
9. Cellular automata for simulating lava flows: A method and examples of the Etnean eruptions / D. Barca [et al.] // Transport Theory and Statistical Physics. – 1994. – Vol. 23, № 1–3. – P. 195–232.
10. Di Gregorio, S. An empirical method for modeling and simulating some complex macroscopic phenomena by cellular automata / S. Di Gregorio, R. Serra // Future Generation Computer Systems. – 1999. – № 16. – P. 259–271.
11. Pyroclastic flows modeling using cellular automata / M. V. Avolio [et al.] // Computers & Geosciences. – 2006. – № 32. – P. 897–911.
12. Developing an effective 2-D urban flood inundation model for city emergency management based on cellular automata / L. Liu [et al.] // Nat. Hazards Earth Syst. Sci. – 2015. – № 15. – P. 381-391.
13. A Cellular Automata Fast Flood Evaluation (CA-ffé) Model / B. Jamali [et al.] // Water Resources Research. – 2019. – № 55. – P. 4936–4953.
14. Cirbus J. Cellular Automata for the Flow Simulations on the Earth Surface, Optimization Computation Process / J. Cirbus, M. Podhoranyi // Appl. Math. Inf. Sci. – 2013. – Vol. 7, № 16. – P. 2149–2158.
15. Application of cellular automata approach for fast flood simulation / B. Ghimire [et al.] // CCWI 2011: Computing and Control for the Water Industry, Exeter, UK, September 5–7, 2011 / University of Exeter. – University of Exeter, 2011.
16. A weighted cellular automata 2D inundation model for rapid flood analysis / M. Guidolin [et al.] // Environmental Modelling & Software. – 2016. – № 84. – P. 378–394.
17. Development of a diffusive wave shallow water model with a novel stability condition and other new features / M. Jahanbazi [et al.] // Journal of Hydroinformatics. – 2017. – Vol. 19, № 3. – P. 405–425.
18. Néelz, S. Desktop review of 2D hydraulic modelling packages / S. Néelz, G. Pender. – Bristol : Environment Agency. – 2009. – 63 p.
19. Néelz, S. Benchmarking the latest generation of 2D hydraulic modelling packages / S. Néelz, G. Pender. – Bristol : Environment Agency. – 2013. – 194 p.
20. A highly efficient 2D flood modelling with sub-element topography / S. R. Jameison [et al.] // Proceedings of the ICE – Water Management. – 2012. – Vol. 165, № 10. – P. 581–595.
21. Petrov, D. O. Algorithm for calculating the boundaries of the flooded area for a river network with modeling the propagation of water using a raster representation of the relief / D. O. Petrov, A. A. Volchek, D. A. Kostjuk // BSUIR reports. – 2016. – № 5 (99). – P. 73–78.
22. A system for calculating and visualizing a flood zone based on a cellular automaton / D. O. Petrov [et al.] // Collection of materials of the international scientific and practical conference dedicated to the year of science in the Republic of Belarus: in 2 parts, Brest, September 25–27, 2017 / Institute of Nature Management of the National Academy of Sciences of Belarus, BrSU A. S. Pushkin, BrSTU ; editorial board: A. K. Karabanov [et al.] ; scientific. ed. A. K. Karabanov, M. A. Bogdasarov. – Brest : BrSU, 2017. – Part 1. – P. 145–148.
23. Petrov, D. O. Evaluation of the adequacy of the use of geometric methods for constructing a flood zone for flood plain rivers / D. O. Petrov, A. A. Volchek // Tourist and natural potential of water bodies of the Belarusian-Polish border area: materials of scientific-practical conference, Brest, October 30–31, 2020 / Ch. Ed. N. V. Mikhalchuk. – Brest : Alternative, 2020. – P. 127–130.

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