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CURVATURE MATTERS: UNRAVELING THE EARTHQUAKE RESPONSE OF BOX-GIRDER BRIDGES

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Abstract

The paper proposes a simpler technique for producing an idealization of curved box-girder type bridge decking under dynamic stresses produced by earthquake related excitation. The analysis focuses on two types of bridge decks with curved cellular structures, examining both single-cell and multiple-cell configurations. This approach may be used to rectangular and trapezoidal box-girder sections with both equal and unequal dimensions of cells. The proposed element "Panel Element (PE)" is coded as Component Element (CE) and has proved to be capable of modeling a full plane panel of a curved cellular deck in its three-dimensional behavior by one element only. For verification purpose and to demonstrate the range of applicability of the new idealization technique, a comparative study was made with the Finite Element Method (FEM), as a standard procedure, used to idealize the box-girder bridge decks. Different configurations of curved box-girder bridge decks are considered to provide a thorough understanding of the dynamic behavior of the curved bridge deck when acted upon by earthquake-based excitation besides the validation purposes. A computer program using (MATLAB R2012b) is specially written using the proposed algorithm of the new idealization technique to evaluate the earthquake analysis results. Comparison was made with those evaluated by the finite element approach using the ready software (ANSYS 12.0) to check the adequacy and suitability of the proposed element in analyzing the box-girder concrete bridge decks. The results showed that the Panel Element Method (PEM) has proved to be valid in estimating the earthquake response for both cases of single and double cell bridge decks, for all the ranges of the aspect ratios; the results obtained by the Panel Element Method (PEM) are acceptable, with an error of less than (12 %) in deflection and less than (18 %) in moments and shear forces for the cases of very large aspect ratios. This research demonstrates the validity of the proposed method "Panel Element Method (PEM)" with wide range of applicability for the dynamic behaviors of free and forced vibration response analysis and the approximate earthquake response analysis of the curved box-girder type of bridge decks of different configurations.

Keywords: curved box-girder bridges, seismic response, panel element method, finite element method, box girder, box-girder bridges.

КРИВИЗНА ИМЕЕТ ЗНАЧЕНИЕ: РАСКРЫВАЯ СЕЙСМИЧЕСКИЙ ОТКЛИК СОТОВЫХ МОСТОВ

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Реферат

В статье предлагается упрощенная методика идеализации криволинейных сотовых пролетных строений мостов при динамических нагрузках, вызванных сейсмическими воздействиями. Анализ сосредоточен на двух типах пролетных строений с криволинейными ячеистыми структурами: одно- и многосоточных конфигурациях. Данный подход может быть применен к прямоугольным и трапецеидальным ячеистым сечениям с ячейками как равных, так и неравных размеров. Предложенный элемент «Панельный элемент» программно реализован в виде компонентного элемента и доказал свою способность моделировать полноразмерную панель криволинейного сотового настила в трехмерном поведении всего одним элементом. С целью верификации и демонстрации диапазона применимости новой методики идеализации было проведено сравнительное исследование с методом конечных элементов (МКЭ), который используется в качестве стандартной процедуры для идеализации сотовых пролетных строений. Рассматривались различные конфигурации криволинейных сотовых мостовых настилов для получения всестороннего понимания динамического поведения криволинейного пролетного строения при сейсмических воздействиях, помимо целей валидации. С использованием предложенного алгоритма новой методики идеализации была специально написана компьютерная программа на (MATLAB R2012b) для оценки результатов сейсмического анализа. Проведено сравнение с данными, полученными методом конечных элементов с использованием готового программного обеспечения (ANSYS 12.0), для проверки адекватности и пригодности предложенного элемента для анализа сотовых железобетонных пролетных строений. Результаты показали, что метод панельных элементов (МПЭ) доказал свою состоятельность в оценке сейсмического отклика как для одно-, так и для двухсотовых пролетных строений во всем диапазоне значений коэффициентов форм; полученные с помощью метода панельных элементов (МПЭ) результаты приемлемы, с погрешностью менее (12 %) для прогибов и менее (18 %) для моментов и поперечных сил в случаях очень больших коэффициентов форм. Данное исследование демонстрирует обоснованность предложенного метода «Метода панельных элементов (МПЭ)» с широким диапазоном применимости для анализа динамического поведения при свободных и вынужденных колебаниях, а также для приближенного анализа сейсмического отклика криволинейных сотовых пролетных строений различных конфигураций.

Ключевые слова: криволинейные сотовые мосты, сейсмический отклик, метод панельных элементов, метод конечных элементов, коробчатая балка, сотовые мосты.

Introduction

The safety of highway bridges is essential to the sustainable economic prosperity of our community. Highway bridge shapes may vary greatly depending on the practical requirements of the project. Curved bridges are commonly selected over linear bridges due to their lower cost and ease of construction. (Tao and Guan, 2023) [1]. Curved bridges are often observed in urban and mountainous areas due to their favorable compatibility with the surrounding terrain and their capability to permit grade crossings and overpasses with limited space (Kahan et al., 1996) [2]. Furthermore, in order to mitigate the build-up of driver fatigue resulting from driving on lengthy and straight bridges with similar conditions, an increasing number of curved bridges are being employed as substitutes for long linear bridges. Curved bridges are occasionally constructed to harmonize with the surrounding environment for aesthetic reasons. Curved overpass bridges are increasingly favored as a solution to the city's significant traffic problems (Tao and Guan, 2023) [1]. Curved bridges of different kinds have a significant impact on managing urban traffic. Nevertheless, these bridges are more vulnerable to damage caused by earthquakes compared to traditional bridges, mostly due to their irregular shape and uneven distribution of weight. Research has demonstrated that the spatial asymmetry of ground movements would negatively impact curved bridges.

The cellular cross section of the curved box-girder bridge (Figure 1) increases its efficiency by resisting the high torsional moment. Aesthetics, economy, stability, efficacy, and utility all contribute to the curved bridge's widespread recognition. Circular plans with transition curves are common when choosing these. Since curved bridges in the plan experience both bending and torsion due to the curvature of the girders, their analysis is more involved than that of straight bridges. According to Agarwal et al. (2023) [3], a curved bridge's optimal section for design should possess strong torsional stiffness.

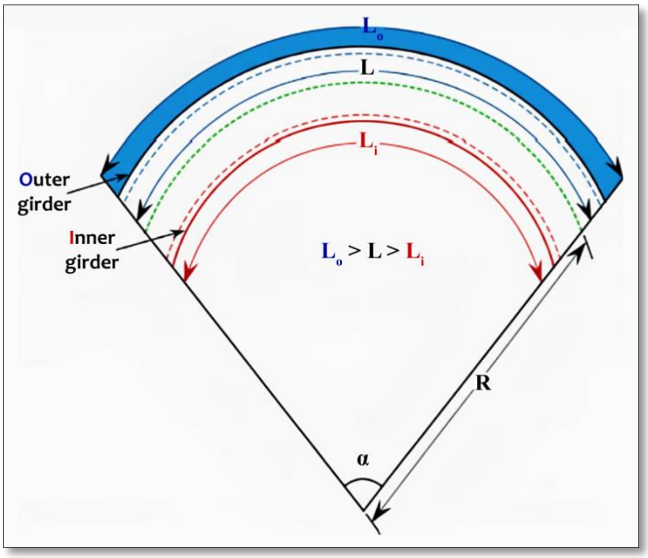


Figure 1 – Curved box-girder bridge deck (Agarwal et al., 2023) [3]

Literature review

Few studies have been conducted on the seismic behavior of curved bridges subjected to ground motion exhibiting spatial variation. Sextos et al. (2004) [4] examined the seismic behavior of curved bridges subjected to multi-support excitation. They achieved this by creating synthetic multi-support ground motion. A study conducted by Burdette et al. (2008) [5] examined the impact of incoherence and traveling waves on the seismic response of both straight and curved bridges. Using the finite element-based software ANSYS, Fangping and Jianting (2012) [6] studied the impact of curvature on prestressed tendons deformation in curved bridges. The mid-span deflections were assessed by modeling and analyzing five distinct curved bridges. Cho et al. (2013) [7] demonstrated the load distribution in straight prestressed concrete (PSC) girder bridges using Finite Element Method (FEM). The GDF was evaluated against the AASHTO LRFD, AASHTO standard factors, and the findings of finite element calculations. Formulas have been proposed to predict the distribution of live loads in PSC girder bridges for preliminary design purposes. Chen and Wang (2014) [8] studied the behavior of curved girder bridges under spatially varying ground motion, with special emphasis on studying random vibration angles. Cheng et al. (2016) [9] used shaking table tests to show how important it is to think about the multi-support excitation effect when analyzing irregular high-rise curved girder bridges for earthquakes. Bahadur et al. (2017) [10] studied the stresses and deflections of a curved, rectangular plate that had only one side supporting it in 2017 and observed how the curve angle, span-thickness ratio, and aspect ratio affected the plate. Curved bridges are more susceptible to travel wave influence, according to shaking table experiments conducted by Li et al. (2017) [11]. Said and Khalaf (2018) [12] looked at a horizontally bent box girder bridge and found the live load moment distribution factor using experimental data that was loaded with AASTHO loads. Agarwal et al. (2019) [13] investigated the maximum bending moment and shear strength in a single cell inclined box girder bridge using finite element analysis. The researchers examined the effect of span, beam spacing, and span depth ratio on a skewed box girder bridge. In their study, Yuan et al. (2022) [14] investigated the mechanical properties and torsional behavior of a curved square-girder bridge. Further finite element analyses have quantified the response of curved girder and box-girder bridges to various loads, examining parameters such as curvature angle and span-depth ratio [10].

More lately, Temimi, et al. (2025), focused on the vibration characteristics of 3D curved box-girder bridges by using the Panel Element Method, and using the finite element method by ANSYS program for many deck types of box-girder bridges under earthquake loads [15].

Collectively, this body of literature confirms that curvature fundamentally alters a bridge's seismic and structural response, necessitating specialized analysis that accounts for both spatial ground motion variation and unique curvature-induced effects.

Research Importance

The majority of curved bridge assessments currently concentrate on static and linear dynamic evaluations, which take into consideration parameters such as natural frequencies, modal shapes, and damping conditions. This analysis aims to suggest recommendations for structural design and spectrum analysis, including support arrangements. The relationship between natural frequency, bridge connections, and dynamic performance in basic box girder curved bridges, as well as the effect of radius curvature, has been studied before (Jeon et al., 2016) [16]. It is important to study how different properties affect the typical shapes of curved bridges.

In present times, design codes often advise examining various seismic inputs and using the highest allowable values for design objectives. Applying this technique is tedious and requires significant computing costs. Creating a more efficient technique for identifying the inputs most susceptible to a curved bridge design is very challenging. It is important to determine the seismic input parameters that are most susceptible to damage and the corresponding main seismic interactions for curved bridges. The investigation focused on the dynamic reactions of curved bridges by the utilization of finite element analysis and parametric studies. A framework for seismic design of a curved bridge was developed based on a thorough investigation by providing a new simplified procedure and an alternative reliable idealization technique for dynamic and earthquake response analysis of curved box-girder type bridge decks.

Scope of Study

This study deals with the dynamic analysis of curved box-girder bridges subjected to earthquake base excitation, which is characterized by two orthogonal components, each of which is perpendicular to the longitudinal axis, but one is assumed to be in the vertical Y-direction and the other in the horizontal transverse X-direction. The 20 February 1990 modified smooth pseudo-acceleration design spectrum of AL-Hindya Earthquake characterizes each base excitation component. Due to the lack of acceleration records in the vertical Y-direction, the same response spectrum that is used in the analysis of bridge decks is acted upon by lateral base excitation, the same spectrum is assumed for the vertical earthquake analysis or using El-Centro components (scaled down). It is also assumed that the motion of all supports of the bridge has phase excitation, that is, all supports are acted upon by the same base excitations simultaneously. Since the box-girder bridges considered in this study are curved in plan, the lack of symmetry results in a coupled response. Therefore, any base excitation component produces a combined response of bending and torsion, even under lateral excitation.

Problem Statement

The increasing demand for reducing traffic congestion has led to the construction of additional highway bridges, particularly curved bridges. There has been much study of the structural performance of curved bridges since they were first designed and built. The structural intricacy of curved bridges causes them to react dynamically differently than straight bridges. Between the 1970-s and 1990-s, many powerful earthquakes resulted in extensive destruction and economic losses for curved bridges in numerous nations globally. Subsequently, various countries have intensified their endeavors in the realm of curved bridge seismic analysis (Tao and Guan, 2023) [1]. Curved bridges have distinct dynamic reactions, particularly in the case of small-curvature bridges, as compared to linear bridges and exhibit a mix of moments and torsion when subjected to both vehicular loads and horizontal seismic forces. When examining the curved bridge's reaction in both the horizontal and vertical orientations, it is difficult to separate the moment-torsion combination. Developing a reasonable theoretical dynamic analysis of a curved bridge is a challenging task due to the curvature and torsional vibration complexity. Because of the significant actions that linear bridges show in these X-Y directions, earthquake input tests that are both horizontal and vertical are generally carried out. Curved bridges are not as resistant to seismic damage as regular bridges, mostly because of their irregular mass distribution and irregular structure. Earlier studies have established that the spatial variation of ground motion has a detrimental impact on curved bridges (Kahan et al., 1996) [2]. Several curved bridges have experienced substantial destruction during earthquakes, but none of the existing seismic specifications – like the 2008 Highway Bridge Seismic Design Specifications [17], the Caltrans Seismic Design Standard (Caltrans 2025) [18], and the American AASHTO Seismic Design Guide (AASHTO 2023) [19] – have made recommendations or used techniques to avoid these issues. This is attributed to a lack of understanding of the dynamic actions of curved bridges during earthquake occurrences. (Wang et al., 2010) [20].

The objective of the present research is to analyze the longitudinal and transverse earthquake motion of the bridge and to determine the design forces and moments at the supports bases by the Finite Element Method (FEM) and the Panel Element Method (PEM). Then a comparative study of the design forces and moments found from these two methods has been made. It is expected that the findings of this study will lead to a better understanding of the behavior of bridges under seismic loading. For simplicity of the analysis, linear material behavior is assumed in this study.

Idealization of Curved Box Girders Using Panel Element Method

A curved box-girder bridge deck of a square or rectangular and trapezoidal in cross-section typically consists of planar units or non-planar units interconnected to each other to form a three-dimensional structural system.

An idealization procedure for modeling box-girder type bridge decks designated as the Panel Element Method (PEM) is proposed in this work. The Finite Element Method (FEM) idealization procedure, as used in this study for validation purposes.

The Panel Element Method (PEM) Idealization Method

This paper presents the Panel Element Method (PEM), an idealization approach for panel elements (PE). Because the fundamental component is derived via altering an element utilized in an existing comparable frame technique, this process might be categorized as an analogous frame approach.

The derivation and modification of the Panel Element Method (PEM) take into consideration the following assumptions that are found to be necessary for the formulation of the problem.

1. The bridge is assumed to be an assemblage of a finite number of flat plates or wall panels.
2. Each wall or slab panel is modeled by the conventional space frame element.
3. The analysis takes into account the flexural and shear deformations of each individual panel element (PE) in the plane.
4. Every panel element's (PE) out-of-plane flexural and shear deformations are also taken into account.
5. Diaphragms are regarded as eternally unyielding inside their own planes and flexible when positioned outside of those planes.
6. A partially rigid interface is assumed between panel elements and diaphragm in both X-Y directions, that is; relative rotational, is allowed between the panel element and the diaphragm only in the plane of panels with no distortion of the cross-section.

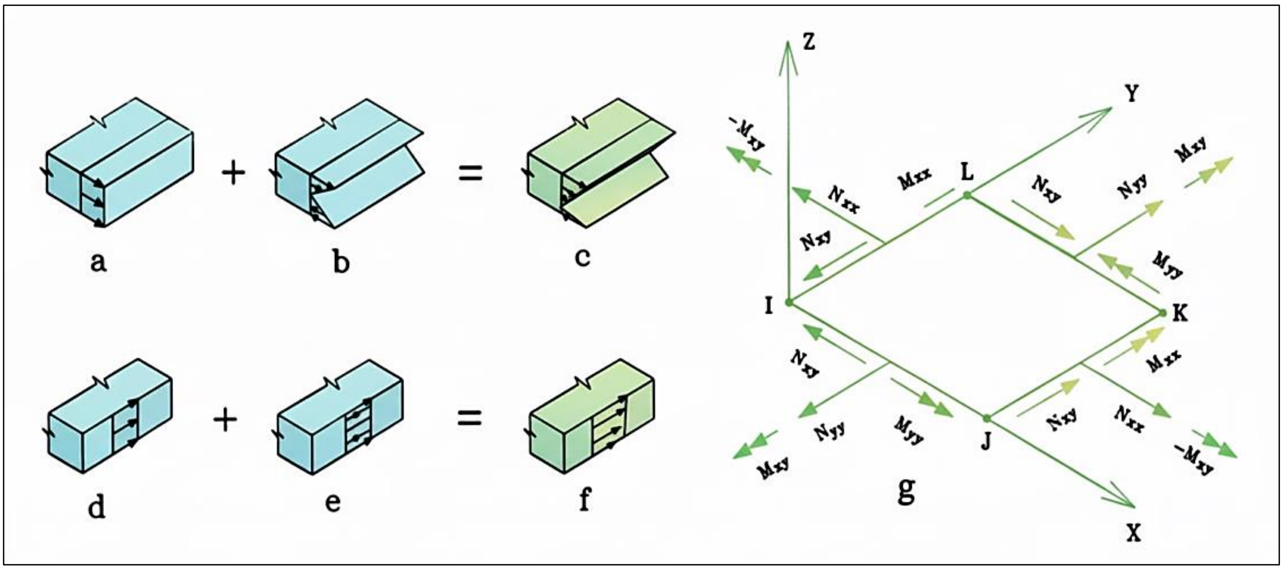
Finite Element Idealization Method

The present work employs the finite element idealization process to verify the efficacy of the suggested Panel Element Method (PEM) and to facilitate comparisons. The finite element idealization approach is employed for the analysis utilizing the ANSYS12 program and the CivilFEM12 software for ANSYS12.

A box-girder bridge deck normally consists of top and bottom slabs, vertical webs, and transverse members (rigid diaphragms). All these parts are modeled by an assemblage of the general four nodes flat shell element as shown in Figure 2.

Transverse members, also known as rigid diaphragms, are assumed to be rigid in one plane and flexible in another. This prevents the in-plane degrees of freedom (d. o. f.) of the transverse member's nodes from being slavishly controlled by a master set of in-plane d. o. f. defined at the mass center of the box-girder bridge deck cross-section. However, all degrees of freedom which are not in-plane of diaphragms are not constrained to this transformation.

For the numerical examples of earthquake response analysis that were considered, a specific mesh size is chosen to represent the building at the outset, and then the mesh is fine-tuned until two consecutive solutions achieve a maximum difference of less than 2 %. This process guarantees convergence and accurate results.



a) axial stress; b) bending stress; c) total in plane stress; d) shear stress; e) twisting stress; f) total shear stress; g) stress conventions for thin plane shell element (Iraq Specification, 1978) [21]

Figure 2 – Three-dimensional plane shell element

Dynamic Analysis of Curved Box-Girder Bridges Subjected to Earthquake

In the dynamic analysis, the main techniques currently used.

Response Spectrum Techniques

This approach is founded on the mode superposition methodology. The general procedure is to compute the response of each of the structure's individual modes and then to combine these responses to obtain the overall response. Only a few modes can be included in computing any particular response of the system. The specific modes, which must be considered, depend upon the properties of the structures and the particular quantity, which is being computed, that is, the modal mass participation factors in each mode.

Direct Integration Techniques

The direct integration of the equations of motion is derived through a systematic and sequential process. This approach is applicable to any seismic motion on a structure, wherein moment and force diagrams may be obtained at specified intervals throughout the applied motion for both linear elastic and nonlinear elastic material behavior. This method is the more nearly complete dynamic analysis technique so far devised and is unfortunately correspondingly expensive to carry out.

Normal Mode Technique

The normal mode technique is a more limited approach than direct integration, as it depends on an artificial combining of the forces and displacements associated with a chosen number of them using modal superposition.

Equation of Earthquake Motion

In a typical dynamic problem, the motion of a system excited by an external dynamic load, and the complete set of forces acting on the system in addition to the external forces are inertia, damping and elastic forces which resist the motion and are proportional to the accelerations, velocities and displacements of the system, respectively. Thus, the equation of dynamic equilibrium of all forces acting on a multi-degree of freedom system at any time (t) can be written as follows

$$\left[\mathrm{M}\right]\cdot\left\{\ddot{\mathrm{U}}\right\}+\left[\mathrm{C}\right]\cdot\left\{\dot{\mathrm{U}}\right\}+\left[\mathrm{K}\right]\cdot\left\{\mathrm{U}\right\}=-\left[\mathrm{M}\right]\cdot\left\{\mathrm{R}\right\}\cdot\ddot{\mathrm{U}}_g . \quad (1)$$

In which [M], [C] and [K] denote, respectively, the mass, damping and stiffness matrices of the bridge deck, corresponding to the structure degrees of freedom {U}; the variable {R} represents an earthquake effect vector that is composed of both positive and negative values. Positive values indicate the degree of freedom in the direction of the base excitation component, while negative values indicate other directions. The vibration mode of the structure may be obtained by transforming Equation (1) into the normal coordinates, that is

$$\ddot{\mathrm{Y}}_n+2\zeta\omega_n\cdot\dot{\mathrm{Y}}_n+\omega_n^2\cdot\mathrm{Y}_n=\frac{\ell_{nr}}{\mathrm{M}_n}\cdot\ddot{\mathrm{U}}_{gr}\left(t\right), \quad (2)$$

where $M_n=\Phi_n^T . M . \Phi_n$ is the generalized mass at mode (n);

$l_{nr}=\Phi_n^T . M . R_n$ is the modal earthquake excitation factor in the r-direction (r = X, Y).

And $\ddot{\mathrm{U}}_{\mathrm{gr}}\left(\mathrm{t}\right)$: The time varying base acceleration component in the r-direction. The solution of the equation (2) may be written as

$$\mathrm{Y}_n\left(t\right)=\frac{\ell_n}{\mathrm{M}_n\cdot\omega_n}\cdot V_n\left(t\right), \quad (3)$$

where, $V_n(t)$ the earthquake-response integral, defined as

$$V_n(t)=\int_0^t\ddot{\mathrm{U}}_g(\tau)\cdot\exp\left[-\zeta\omega_n(t-\tau)\cdot\mathrm{Sin}\,\omega_n(t-\tau)\right]d\tau . \quad (4)$$

If the response spectrum of the ground motion is available, the maximum response of the system at each node can be obtained from it, depending upon the natural time-period and the damping ratio of the structure. The response could be spectral acceleration, velocity, or displacement, for which

$$V_{\max}(\zeta,\mathrm{T})=\frac{1}{\omega}\cdot S_a(\zeta,\mathrm{T}), \quad (5)$$

where T: Natural period, and S_a: the spectral acceleration or more properly, the spectral Pseudo-acceleration because it is not exactly the peak acceleration value in general, denoting the maximum acceleration of the structure relative to the ground.

Then, substituting equation (5) into equation (3) gives

$$\mathrm{Y}_{n,\max}=\frac{\ell_n}{\mathrm{M}_n\cdot\omega_n^2}\cdot S_a(\zeta,\mathrm{T}) . \quad (6)$$

The calculated displacements (U_n) may be determined by multiplying the mode shape (Φ_n) with the generalized coordinate amplitude (Y_n). Therefore, the total displacements can be expressed as follows

$$\left\{U_{nr}\right\}=\Phi_n\cdot\frac{\ell_{nr}}{\mathrm{M}_n}\cdot\frac{S_{a_r}(\zeta,\mathrm{T})}{\omega_n^2} . \quad (7)$$

The elastic forces (Fs) corresponding to the relative displacements may be derived by directly multiplying the relative displacements by the stiffness matrix, such that

$$\mathrm{F_s}\left(t\right)=\mathrm{K}\,.\,\mathrm{U}\left(t\right)=\mathrm{K}\,.\,\Phi\,.\,\mathrm{Y}\left(t\right). \quad (8)$$

Expressing these forces in terms of the corresponding inertia forces created in the undamped vibration is a more convenient approach (Clough and Penzien, 1993) [22] such that

$$\left\{F_{S_{nr}}\right\}=\omega_n^2\cdot\left[\mathrm{M}\right]\cdot\left\{U_{nr}\right\}=\left[\mathrm{M}\right]\cdot\left\{\Phi_n\right\}\cdot\frac{\ell_{nr}}{\mathrm{M}_n}\cdot S_{a_r}(\zeta,\mathrm{T}) . \quad (9)$$

Structural Response

It should be pointed out that, in practice, the superposition of the mode responses is usually done in one of three ways (Dilger et al., 1988) [23]. The most conservative approach that yields an upper limit is to numerically add the response of the modes. This approach yields reasonable results for cases where the contribution of the fundamental mode predominates. For many problems this is usually true.

A less conservative approach is to take the sum of the fundamental mode response plus the square root of the sum of the squares of the higher modes. This will yield more reasonable results for cases where the contributions of the higher modes are appreciable.

A third approach is to obtain a total maximum response by taking the root mean square, that is, the square root of the sum of the squares of the maximum responses. The third approach is considered in the present study; thus, the maximum forces are approximated by

$$F_{S_{\max}}=\sqrt{(F_{S_1})_{\max}^2+(F_{S_2})_{\max}^2+\cdots\cdots\cdots\cdots+(F_{S_n})_{\max}^2} , \quad (10)$$

where ($F_{S1\max}$, $F_{S2\max}$, …, $F_{Sn\max}$) are calculated from equation (9).

The resulting force vector corresponds to the free degrees of freedom (d. o. f). The reaction at support is obtained by the static approach, thus

$$\left[\mathrm{K}\right]\cdot\left\{e\right\}=\left\{F_{S_{\max}}\right\}, \quad (11)$$

where [K] denotes the stiffness matrix of the overall structure after applying boundary conditions and {e}, denotes the displacement vector produced by the static analysis of bridge deck subjected to the maximum force vector {$F_{S\max}$}, which are obtained from the response spectrum analysis. Back substitution is applied to evaluate the reactions at supports.

Materials and methods

This study investigates the seismic response of curved box-girder bridge decks using a newly proposed Panel Element Method (PEM), validated against the conventional Finite Element Method (FEM). The research methodology involved numerical modeling and simulation of various curved box-girder bridge configurations.

Computer Programs by MATLAB and ANSYS Software

A computer program was written for the dynamic analysis of the box-girder bridge decks by using the proposed Panel Element (PE) method. Dynamic analysis which has been adopted consists of free and forced vibration (earthquake response analysis).

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The program group of "FSPE-DYNAMIC" is coded in MATLAB language by using a PC-computer. This program is used to analyze the curved box-girder bridge decks for any support condition and under any live load type.

Material properties were defined by an Elastic Modulus of 23,5 x 10^6 kN/m², a Weight Density of 24,517 kN/m³, and a Poisson's Ratio of 0,20. The important material properties of MATLAB models (Linear, Elastic and Isotropic) that are used in the studied cases which are shown in Table 1.

Table 1 – Material properties of MATLAB program

No.	Material Properties	Values
1	Elastic Modulus (E)	23,5 x 106 kN/m²
2	Weight Density (γ)	24,517 kN/m³
3	Poisson's Ratio (υ)	0,20

The finite element modeling and analysis conducted in this work were carried out using program ANSYS. The calculations conducted in this study were executed utilizing ANSYS version 12,0.

The Shell and Beam elements (ANSYS, 2025) [24] that are used in this method of modeling.

1. Shell 63 (elastic shell).
2. Beam 4 (3-D Elastic Beam).

The important material properties of ANSYS models (Linear, Elastic, and Isotropic) that are used in the studied cases are shown in Table 2.

There are primarily four case studies of cross-sectional areas of curved box-girder bridges that are modeled in ANSYS for the current study as shown below.

Description of Case Studies

The case studies which are presented in this work include two types of curved bridges, two types of box-girder deck bridge, and two types of cross-sections of the box-bridge. The types of studied curved bridges are as follows: the first type of bridge is a curved bridge with (20 m) span lengths, and acute angle of (20° degree) and radius of curvature (57,3 m), as shown in Figures 3 c and 5 c, while the second type of curved bridge is (30 m) span length, with acute angle of (30° degree) and radius of curvature (57,3 m), as shown in Figures 4 c and 6 c.

The types of studied cellular decks are as follows: the first type is a single cell as shown in Figures 3 d and 5 d, while the second is a double cell curved deck as shown in Figures 4 d and 6 d. The types of studied cross-sections of the box-bridge are as follows: the first type is a rectangular cross-section, as shown in Figures 3 a and 5 a, while the second is a trapezoidal cross-section curved deck, as shown in Figures 4 a and 6 a.

Typical layout and cross-section dimensions for each type are shown in Figures 3, 4, 5, 6 and the material properties are listed in Tables 1 and 2.

These decks are the major decks of an existing bridge at Baghdad city (Ur-Qaherah bridge). For the purpose of the present analysis, it is assumed that each bridge is of a single (simply supported) span of length and central angle shown in Figures 3 c and 4 c. It is worth mentioning that each deck is curved in the longitudinal direction by a central angle (θ = 20° and θ = 30°) and in the vertical direction by the profile described in Figures 3 b and 4 b. All bridge decks, which are studied in the following numerical case studies, are assumed to be of reinforced concrete, which is assumed to be a linear elastic material. The required material properties in the analysis are shown in Tables 1 and 2.

Table 2 – Material properties of ANSYS models

No.	Material Properties	Values
1	Elastic Modulus (E)	23,5 x 103 N/mm²
2	Weight Density (γ)	2,500 kg/m³
3	Poisson's Ratio (υ)	0,20

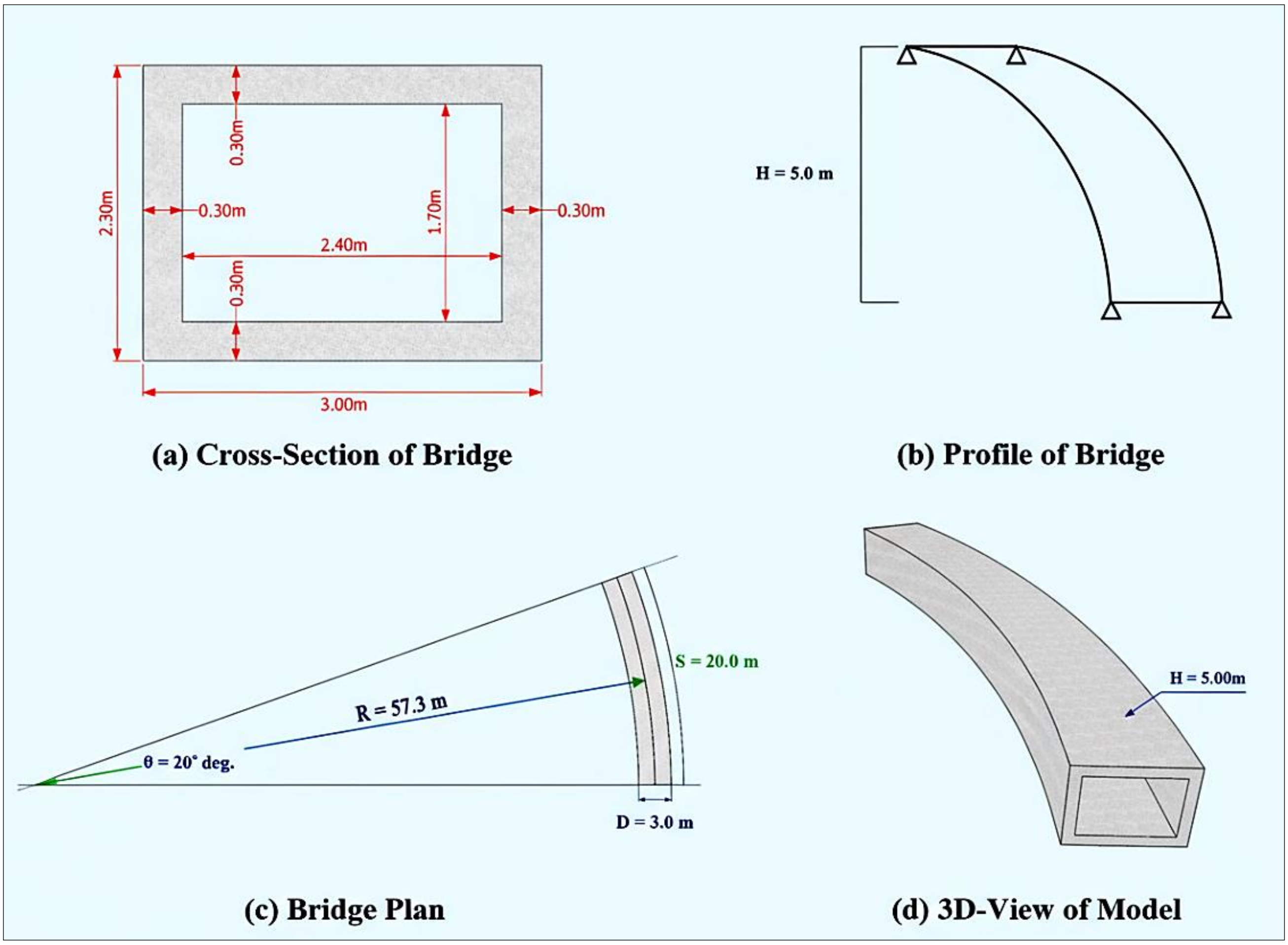

Figure 3 – Case Study No. 1. Rectangular single cellular curved box-girder bridge

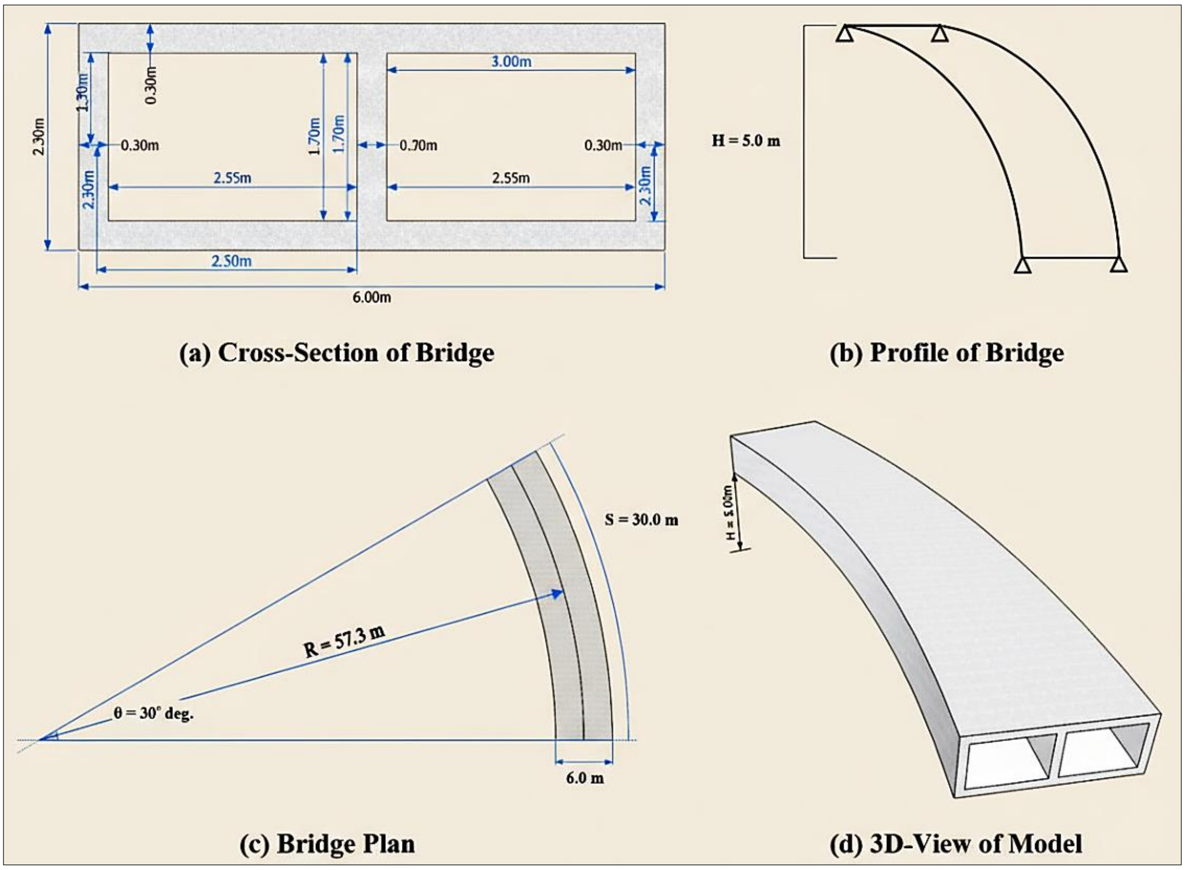

Figure 4 – Case Study No. 2. Rectangular double cellular curved box-girder bridge

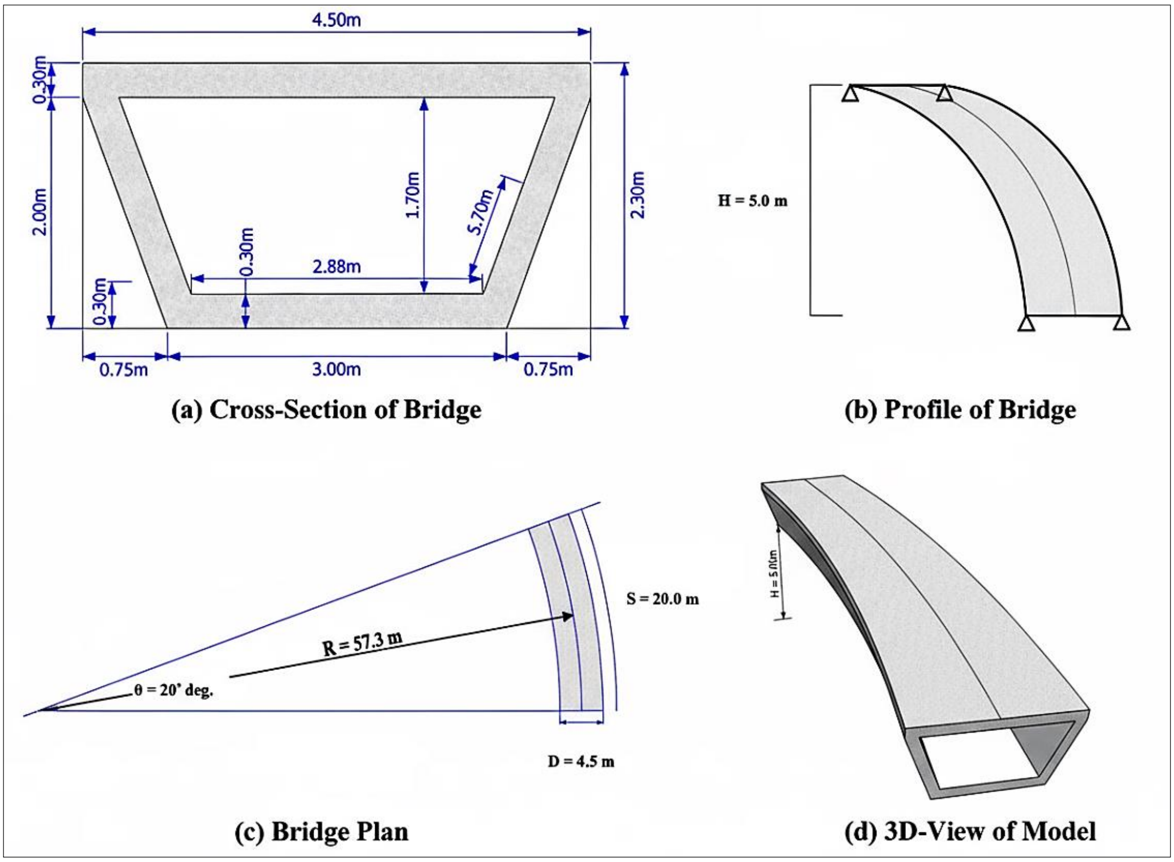

Figure 5 – Case Study No. 3. Trapezoidal single cellular curved box-girder bridge

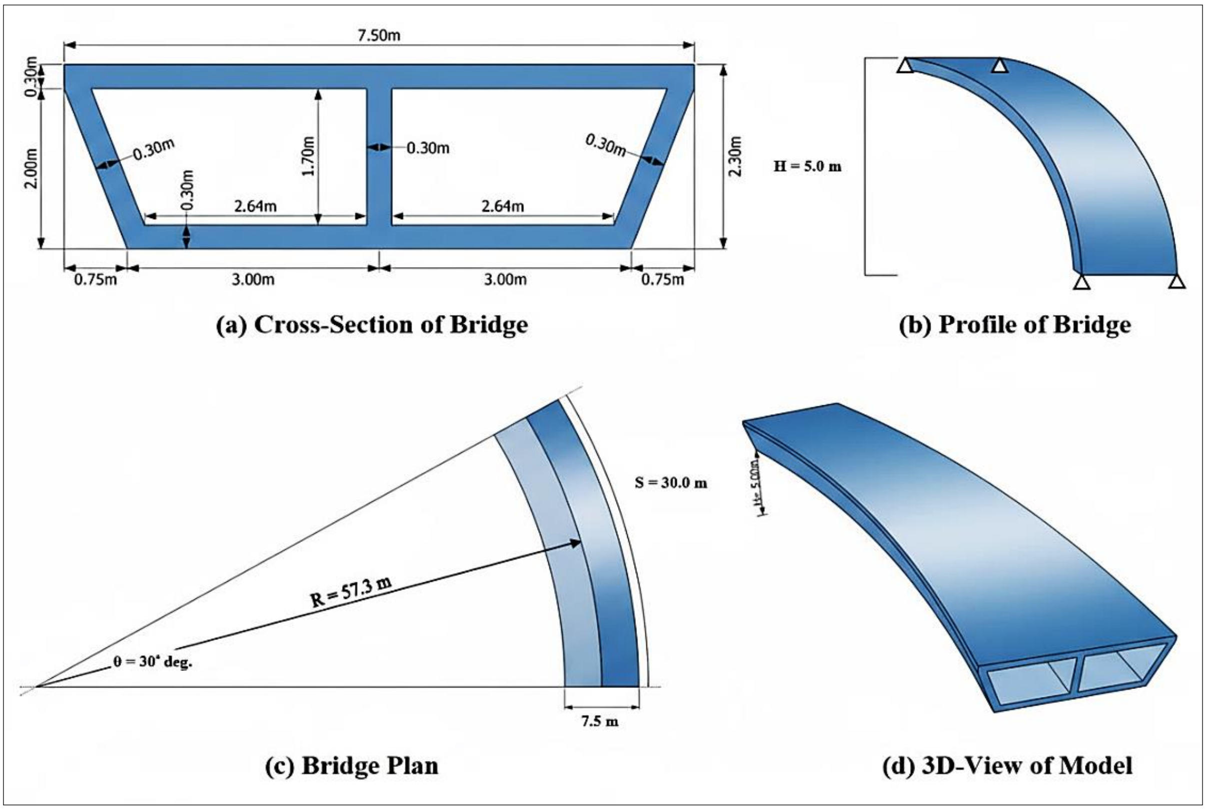

Figure 6 – Case Study No. 4. Trapezoidal double cellular curved box-girder bridge

Numerical Case Studies

The bridge decks were analyzed for their earthquake response when acted upon by a lateral base excitation (orthogonal to the longitudinal tangential axis of the bridge at its mid-span) and a vertical base excitation. All these bridges are analyzed using both the proposed Panel Element (PE) approach and the results are compared to those obtained by the Finite Element (FE) procedure using the ready software ANSYS. For more details see (Al_Temimi F., 2014) [25].

The resulting moments and shear forces of each case study are given in two ways and as follows:

1) absolute response that is, the maximum moment and maximum shear force;
2) normalized response, that is, the resulting moment and shear forces are normalized to:
   a) total mass of the bridge (m*);
   b) the product of (m*) times the span length of the bridge (L).

Research Results

Some parametric studies are carried out to provide a better idea about the behavior of curved box girder bridges including four main factors of: number of cells, web to flange thickness ratios, number of diaphragms and live load effects, to provide a better idea about the behavior of curved box girder bridges. The comparative analysis between PEM and FEM demonstrated the validity and efficiency of the proposed method. The results, summarized across multiple parametric studies, are as follows.

Effects of Number of Cells Variation

Two types of cell bridges (single and double) are considered for verification purposes. A cantilever deck of (20 m) span length, (2,3 m) depth and (3 m) width of each cell and element thicknesses of (0,3 m) is studied for their earthquake response.

The absolute and normalized moments, shear forces and deflections of a cantilever deck are given in Tables 3 and 4 for base excitation in (X and Y-directions), respectively.

Analysis showed that a maximum difference of less than (12 %) in deflection, base shears and bending moments is encountered between the proposed Panel Element Method (PEM) of analysis and the finite element (FE) procedure of ANSYS software irrespective of the direction of earthquake base excitation.

Table 3 – Maximum response of a cantilever bridge deck of single and double cell (base excitation in lateral X-direction)

No. of Cells	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
1	FEM	614,742	0,244	0,012	75,672	0,030	1,273
	PEM	691,992	0,275	0,014	81,152	0,032	1,312
2	FEM	705,600	0,510	0,026	92,369	0,067	1,639
	PEM	750,600	0,543	0,027	95,969	0,069	1,837

Table 4 – Maximum response of a cantilever bridge deck of single and double cell (base excitation in vertical Y-direction)

No. of Cells	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
1	FEM	956,133	0,380	0,019	315,773	0,126	4,077
	PEM	1023,383	0,407	0,020	325,925	0,130	4,369
2	FEM	1016,064	0,735	0,037	404,571	0,293	5,535
	PEM	1056,912	0,764	0,038	407,235	0,295	5,700

Effects of Web to Flange Thicknesses Ratio

The bridge decks are studied here for their earthquake response characteristics. A single cell deck is considered with a depth of (2,3 m) and a width of (3,0 m), then a double cell deck is considered with a depth of (2,3 m) but a width of 6,0 m (3,0 m for each cell). The bridges under consideration are of (20 m) single span with partially and fully restrained supports, the radius of curvature of the bridge span is (57,3 m). The thickness ratios varied from (0,5 to 2,0). To demonstrate the range of applicability of the proposed idealization procedure of the Panel Element Method (PEM) to different ranges of (web thickness: Slab thickness) ratio. The single cell bridge deck fully restrained at supports which is discussed in chapter five is used here for earthquake response in two directions (X and Y directions) transverse to the longitudinal axis of the deck (Z-axis).

The results are presented in Figures 7 and 8 for the cases of a single cell and double cell bridge deck. Results reveal that the proposed idealization procedure of the Panel Element Method (PEM) works well in evaluating the response of bridge decks subjected to earthquake base excitation as compared with the Finite Element Method (FEM) with errors not more than (10 %) in the deflection and no more than (17 %) in moments and shear forces when the (tw/ts) thickness ratio reaches (2).

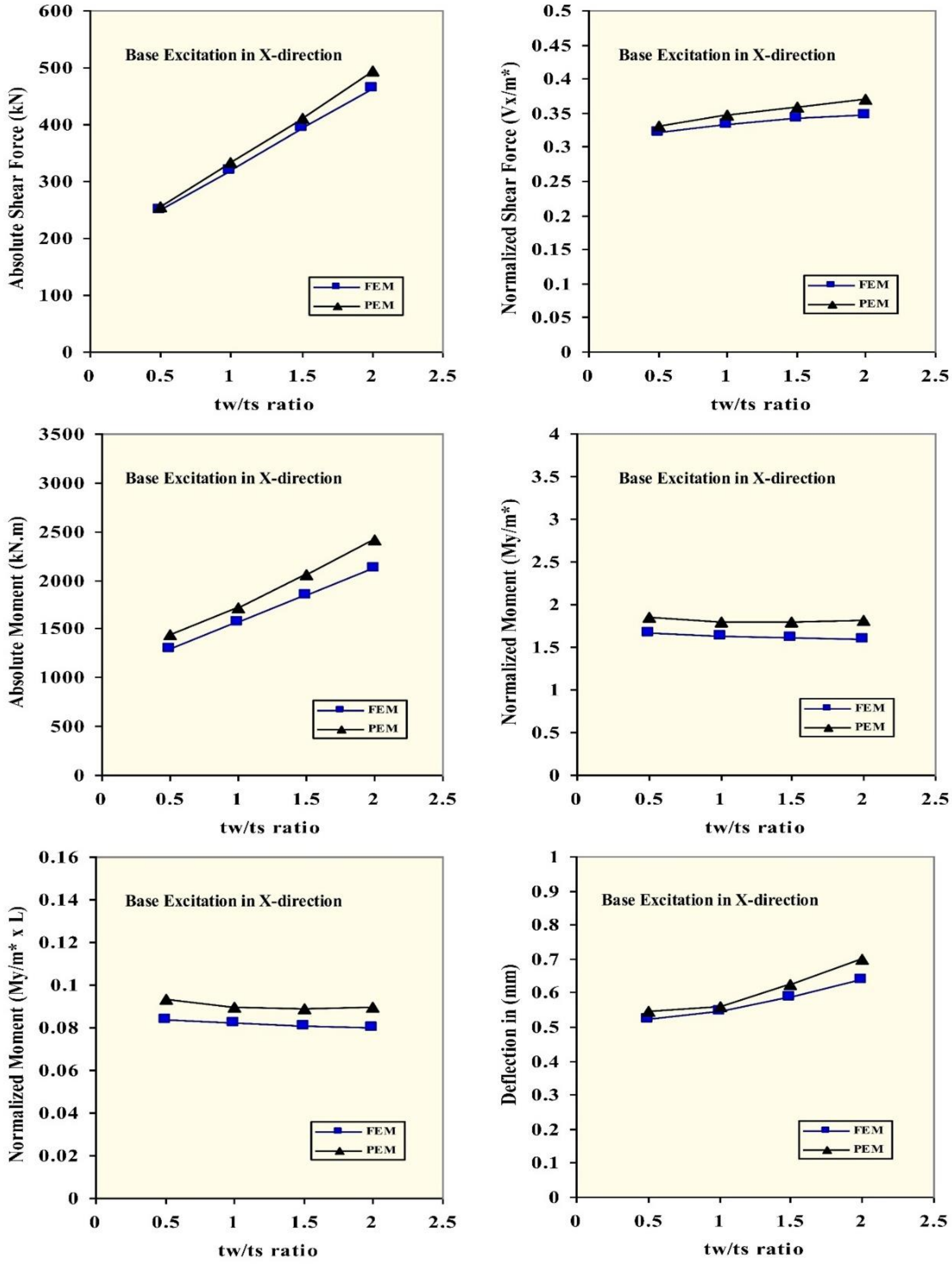

Figure 7 – Maximum response variation with (web thickness: slab thickness) ratio for single curved bridge deck fully restrained at supports (base excitation in lateral X-direction)

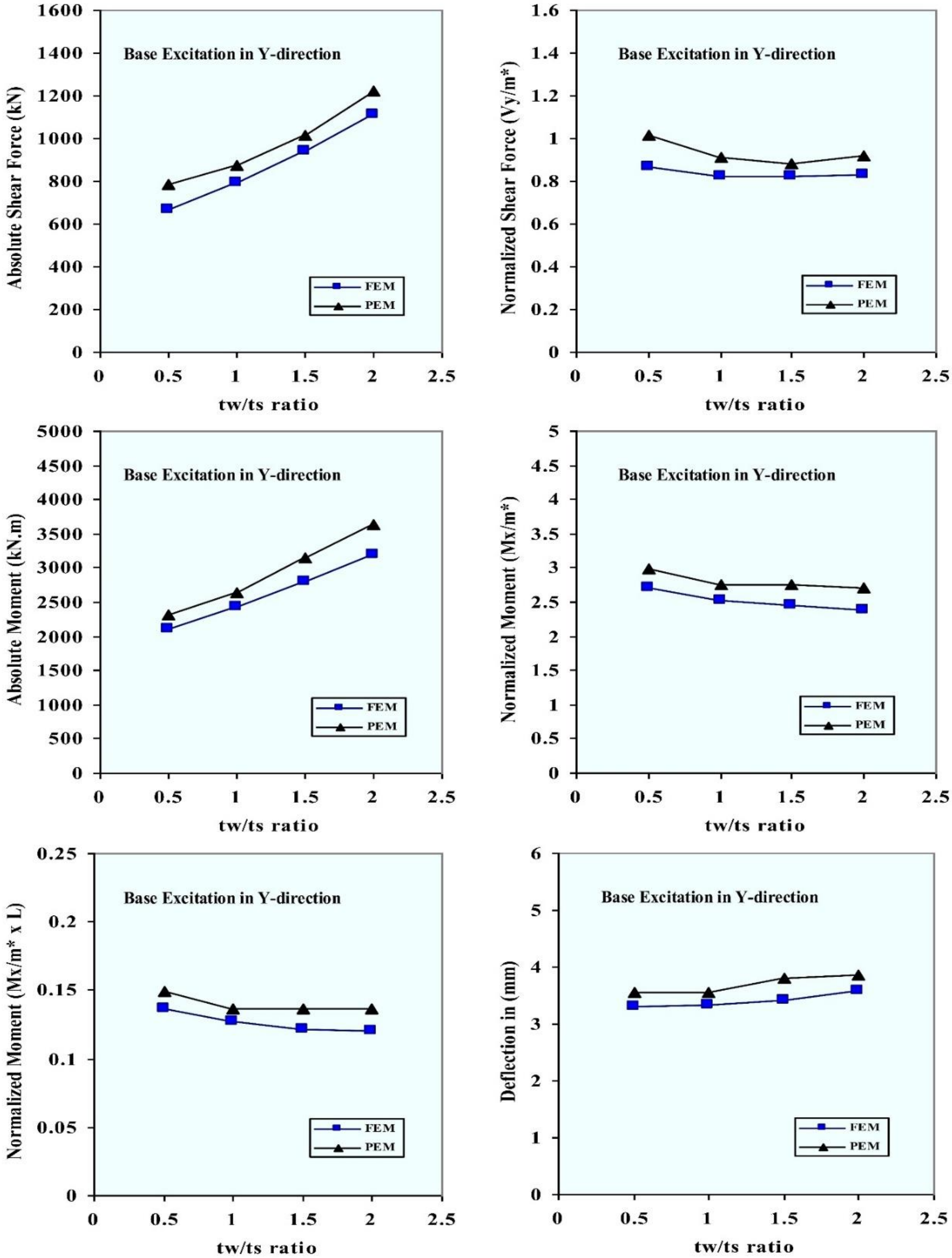

Figure 8 – Maximum response variation with (web thickness: slab thickness) ratio for single curved bridge deck fully restrained at supports (base excitation in vertical Y-direction)

Effects of Number of Diaphragms

Next is a parametric study on the effect of number of diaphragms along the constant span length on the earthquake response behavior of curved bridges to demonstrate the range of applicability of the proposed panel element (PE) idealization scheme for two cases of earthquake base excitation, namely, in a lateral X-direction normal to mid-span tangent and in the vertical Y-direction. The numbers of panels are changed from (2 to 10). In this case, the number of diaphragms represents the number of panels where each panel represents a segment between two diaphragms.

Maximum moments and deflections at mid-span and maximum shear forces at supports as a response of the bridge deck when acted upon by base excitation are shown in Tables 5, 6, 7 and 8, for both partially and fully restrained support conditions for earthquake base excitation in X and Y-directions, respectively.

It can be seen clearly from the results that variation of number of panels results in significant change in the value of the deflection for both X and Y directions and boundary condition types, with errors not more than (12 %) in the deflection and no more than (18 %) in moments and shear forces when the number of diaphragms reaches (10). It is also concluded that the proposed idealization procedure of the Panel Element Method (PEM) is valid for all the range of numbers of diaphragms considered in the study.

Table 5 – Maximum response variation with (number of diaphragms: span) ratio for single cell bridgdeck partially restrained at supports (base excitation in lateral X-direction)

No. of Cells	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
2	FEM	3494,16	2,527	0,126	1157,76	0,837	0,931
	PEM	4109,6	2,972	0,149	1343,68	0,972	1,007
4	FEM	3229,2	2,335	0,117	1285,74	0,930	0,898
	PEM	3799,2	2,748	0,137	1507,04	1,090	0,957
6	FEM	3600	2,603	0,130	1494,36	1,081	0,980
	PEM	4022,4	2,909	0,145	1725,408	1,248	1,040
10	FEM	3895,2	2,817	0,141	1598,94	1,156	1,053
	PEM	4233,96	3,062	0,153	1778,544	1,286	1,092

Table 6 – Maximum response variation with (number of diaphragms: span) ratio for single cell bridge deck fully restrained at supports (base excitation in lateral X-direction)

No. of Cells	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
2	FEM	2088,6	1,510	0,076	649,2	0,469	0,515
	PEM	2459,4	1,779	0,089	753,2	0,545	0,564
4	FEM	2082,24	1,506	0,075	689,04	0,498	0,558
	PEM	2401,92	1,737	0,087	813,16	0,588	0,591
6	FEM	2314,08	1,674	0,084	741,96	0,537	0,607
	PEM	2600,06	1,880	0,094	877,8	0,635	0,627
10	FEM	2498,4	1,807	0,090	787,8	0,570	0,650
	PEM	2720,4	1,967	0,098	918	0,664	0,653

Table 7 – Maximum response variation with (number of diaphragms: span) ratio for single cell bridge deck partially restrained at supports (base excitation in vertical Y-direction)

No. of Cells	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
2	FEM	3744	2,708	0,135	1379,556	0,998	5,502
	PEM	4425	3,200	0,160	1610,8	1,165	6,194
4	FEM	3542,04	2,562	0,128	1499,2	1,084	6,376
	PEM	4094,64	2,961	0,148	1729,232	1,251	6,830
6	FEM	4046,4	2,926	0,146	1568,34	1,134	7,029
	PEM	4572,18	3,307	0,165	1819,2	1,316	7,326
10	FEM	4432,8	3,206	0,160	1666,8	1,205	7,590
	PEM	4923,72	3,561	0,178	1954,08	1,413	7,755

Table 8 – Maximum response variation with (number of diaphragms: span) ratio for single cell bridge deck fully restrained at supports (base excitation in vertical Y-direction)

No. of Cells	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
2	FEM	2665,32	1,928	0,096	982,4	0,710	2,099
	PEM	3126,8	2,261	0,113	1129,56	0,817	2,313
4	FEM	2506,36	1,813	0,091	998,16	0,722	2,049
	PEM	2966,04	2,145	0,107	1182,06	0,855	2,247
6	FEM	2868,84	2,075	0,104	1055,7	0,763	2,300
	PEM	3243,4	2,346	0,117	1207,08	0,873	2,412
10	FEM	3145,68	2,275	0,114	1122	0,811	2,492
	PEM	3487,32	2,522	0,126	1234,8	0,893	2,534

Effect of Live Load

To explain the effect of live load, simple load cases are considered according to the Iraq's Specifications for Bridge Loading (Iraq Specification, 1978) [21].

1. Lane loading, where loads are distributed uniformly over the deck and knife edge load is considered at mid-span to give the maximum response. This load condition is designated the fast load case (load case I).
2. Military Loading, two classes of this loading are studied as follows:
   a) class 100 (Tracked), one tracked at mid-span. This load condition is designated the second load case (load case II);
   b) class 100 (Wheeled), one wheeled at mid-span. This load condition is designated the third load case (load case III).

The uniformly distributed lane load is considered as an additional mass added to the mass density of the structures. The other types of live loads are considered as lumped masses added to the corresponding degrees of freedom in the horizontal transverse (X-direction) and vertical (Y-direction).

All the results, which represent the dynamic response of the bridge deck subjected to earthquake base excitation in X and Y-directions and for both partially and fully restrained boundary conditions are given in Tables 9, 10, 11 and 12.

It can be seen from the above-mentioned tables that a good agreement with response predicted by the finite element is demonstrated out of these numerical case studies, with errors not more than (10 %) in the deflection and no more than (16 %) in moments and shear forces.

Also, the tables give more evidence of the validity of the proposed idealization procedure of the Panel Element Method (PEM).

Table 9 – Maximum response for different live load cases on a single cell bridge deck partially restrained at supports (base excitation in lateral X-direction)

Load Case No.	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
I	FEM	5210,4	3,768	0,188	958,2	0,693	1,63
	PEM	5731,8	4,145	0,207	1030,224	0,745	1,7
II	FEM	5688,12	4,114	0,206	1068,12	0,772	1,729
	PEM	6022,08	4,355	0,218	1136,16	0,822	1,766
III	FEM	6368,72	4,606	0,230	1207,44	0,873	1,898
	PEM	6915,16	5,001	0,250	1232,04	0,891	2,019

Table 10 – Maximum response for different live load cases on a single cell bridge deck fully restrained at supports (base excitation in lateral X-direction)

Load Case No.	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
I	FEM	2685,6	1,942	0,097	703,44	0,509	0,749
	PEM	3027,96	2,190	0,109	764,604	0,553	0,825
II	FEM	3192,48	2,309	0,115	884,76	0,640	1,013
	PEM	3566,88	2,580	0,129	924,84	0,669	1,095
III	FEM	3877,32	2,804	0,140	1006,56	0,728	1,135
	PEM	4045,68	2,926	0,146	1037,64	0,750	1,234

Table 11 – Maximum response for different live load cases on a single cell bridge deck partially restrained at supports (base excitation in vertical Y-direction)

Load Case No.	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
I	FEM	4883,76	3,532	0,177	1486,44	1,075	7,85
	PEM	5149,76	3,724	0,186	1617,6	1,170	8,642
II	FEM	4907,16	3,549	0,177	1633,68	1,181	8,642
	PEM	5287,24	3,824	0,191	1769,88	1,280	9,137
III	FEM	5026,13	3,635	0,182	1862,64	1,347	9,863
	PEM	5513,64	3,987	0,199	1920,24	1,389	10,655

Table 12 – Maximum response for different live load cases on a single cell bridge deck fully restrained at supports (base excitation in vertical Y-direction)

Load Case No.	Analysis Method	Max. Bending Moment			Max. Shear Force		Max. Deflection (mm)
		Abs. (kN.m)	Nor. to m* (m)	Nor. to m* x L	Abs. (kN)	Nor. to m*	
I	FEM	4620	3,341	0,167	1342,08	0,971	6,684
	PEM	5316,3	3,845	0,192	1383,48	1,001	7,029
II	FEM	4666,8	3,375	0,169	1503,6	1,087	6,798
	PEM	5432,4	3,929	0,196	1542	1,115	7,117
III	FEM	4965,6	3,591	0,180	1656,6	1,198	7,029
	PEM	5744,76	4,155	0,208	1670,4	1,208	7,458

Conclusions

In order to assess the efficiency and accuracy of the proposed idealization procedure designated the Panel Element Method (PEM) for earthquake response analysis of curved box-girder deck bridge structures, a number of examples of case studies are analyzed.

Different configurations of curved box-girder bridge decks are considered to verify the proposed Panel Element Method (PEM) against the Finite Element Method (FEM) for both free and forced vibrations. According to the case studies considered in the present research, the major conclusions are drawn.

1. The Panel Element Method (PEM) of idealizing curved box-girder type bridge decks is verified for the dynamic analysis of earthquake response of almost all-practical deck configurations, which are considered.
2. At the same time, a varied reduction in the number of elements and hence, the degrees of freedom (d. o. f) is gained when using the proposed idealization procedure of panel element (PEM) to model the behavior of the bridge under consideration as compared to the traditional finite element (FE) procedure.
3. Moreover, since the number of degrees of freedom needed by the proposed element is limited, the number of equations and iterations are largely reduced and hence less error is encountered.
4. The Panel Element Method (PEM) has proved to be valid in estimating the earthquake response for both cases of single and double cell bridge decks.
5. For all the ranges of the aspect ratios; the results obtained by the Panel Element Method (PEM) are acceptable, with an error of less than (12 %) in deflection and less than (18 %) in moments and shear forces for the cases of very large aspect ratios. It can be seen that the proposed Panel Element Method (PEM) predicts good estimates of cases of small aspect ratios. The normalized value shows good compatibility in response especially in the values of moments when normalized to the product of total mass and the span of the decks.
6. Variation of the number of diaphragms for a constant span length of a bridge deck results in almost no change in the deflection. The errors encountered in estimating the deflection of bridge decks are inversely proportional to the number of panels. The number of diaphragms has proved to be of insignificant influence on the moment and shear force responses of curved bridge decks when acted upon by earthquake base excitations.

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