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## ANALYSIS OF CALCULATION METHODS AND NUMERICAL SIMULATION OF THE STRESS-STRAIN STATE OF REINFORCED CONCRETE ELEMENTS UNDER LOCAL ACTION OF TENSILE FORCES

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### Abstract

The analysis of methods for calculating the load-bearing capacity of reinforced concrete elements for tearing, given in the domestic technical literature, as well as in the literature countries of near and far abroad, has been conducted. A numerical study of the stress-strain state of the tearing zone under the local action of tensile forces was carried out.

**Keywords:** supporting beam, supported beam, tear-off, finite element analysis, tear-off zone.

## АНАЛИЗ МЕТОДОВ РАСЧЕТА И ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ЖЕЛЕЗОБЕТОННЫХ ЭЛЕМЕНТОВ ПРИ МЕСТНОМ ДЕЙСТВИИ РАСТЯГИВАЮЩИХ УСИЛИЙ

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### Реферат

Проведен анализ методов расчета несущей способности железобетонных элементов на отрыв, приведенных в отечественной технической литературе, а также в литературе стран ближнего и дальнего зарубежья. Выполнено численное исследование напряженно-деформированного состояния зоны отрыва при местном действии растягивающих усилий.

**Ключевые слова:** поддерживающая балка, поддерживаемая балка, отрыв, конечно-элементный анализ, зона отрыва.

### Introduction

Over the last 20 years, both in the Republic of Belarus and in other countries of the post-Soviet era and the Eurasian Union, design standards, including reinforced concrete structures, have been introduced and improved. Taking into account new theoretical and experimental data, as well as developed reliability criteria, changes and addenda were made to a number of standard provisions. However, the analysis of regulatory documents on the calculation and design of reinforced concrete structures, as well as other available sources, showed that the calculation provisions in case of the local action of tensile forces have not changed for more than 70 years.

In the existing technical literature for calculating elements for local action of tensile forces (tear-off), including those in force in the national technical normative legal acts [1] and Eurocodes [2], ensuring the requirements of bearing capacity and serviceability is carried out mainly by direct calculation of transverse reinforcement or by additional constructive measures (installation of structural transverse reinforcement and reduction of the pitch of the main transverse reinforcement). However, the practice of operating reinforced concrete structures shows that this does not meet the requirement for the operational reliability of structures, which manifests itself in the form of their serious damage, especially in load-bearing reinforced concrete structures of buildings and structures – destruction by detachment of reinforced concrete trusses, ridges of gable beams, etc. In this regard, the study and development of a methodology for calculating reinforced concrete structures for tear-off, taking into account the criteria for the reliability of structures, is an urgent task and requires both theoretical and experimental confirmation.

The mechanism of destruction under local tensile forces (tearing) is very similar to the mechanism under local shearing (punching). At the same time, if for the latter continuous experimental and theoretical research is being conducted both in the Republic of Belarus and abroad and changes are constantly being introduced into calculation methods and regulatory

documents, then research under the local action of tensile forces in recent decades has been reduced to studies of various anchor elements which work in the condition of the tear-out from the body of concrete or reinforced concrete elements, the mechanics of which differ from that of tearing-off.

### Analysis of methods for calculating the load-bearing capacity of reinforced concrete elements for tearing

According to construction rules SP 5.03.01-2020 [1], checking the resistance of reinforced concrete elements according to maximum states of bearing capacity under local tension (tear-off) for the case of applying a load to the bottom edge or within the height of the element section (Figure 1) according to clause 8.4.2 is carried out from the condition:

$$F_{Ed} \cdot \left(1 - \frac{d_s}{d}\right) \leq \sum (f_{ywd} \cdot A_{sw}), \quad (1)$$

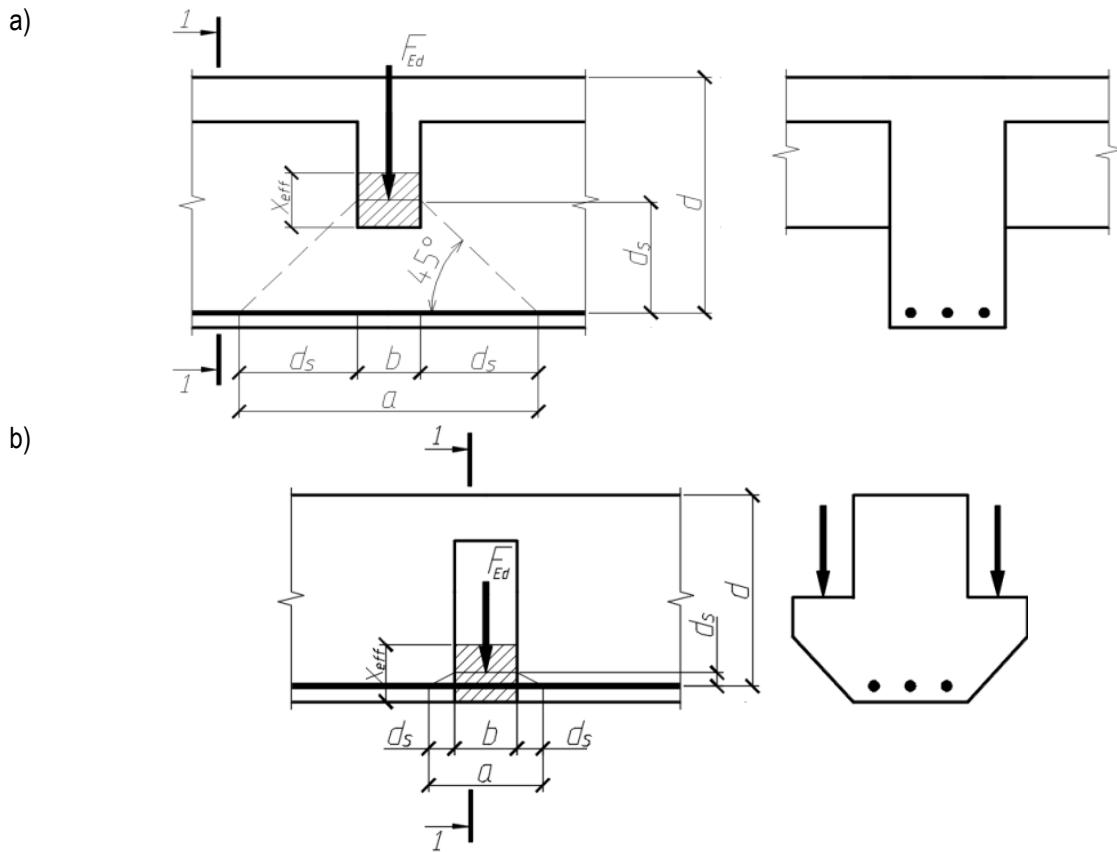
where  $F_{Ed}$  – the calculated value of the tear-off force;

$d_s$  – the distance from the level of transmission of the tear-off force applied to the element to the center of gravity of the longitudinal reinforcement section;

$\sum (f_{ywd} \cdot A_{sw})$  – the sum of transverse forces perceived by clamps installed additionally along the length of the tear-off zone  $a$ , determined by the formula:

$$a = 2 \cdot d_s + b, \quad (2)$$

where  $b$  – the width of the area for transmitting the tear-off force.

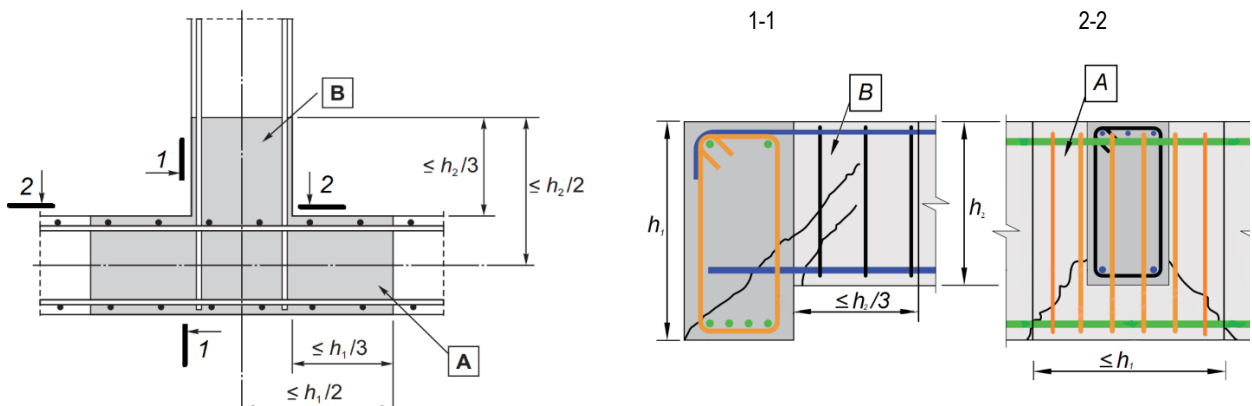


$x_{eff}$  – size of the compressed concrete zone of the supported element  
**Figure 1** – Diagram of determining the length of the tear-off zone, when abutted: a) – beams; b) – consoles [3]

The values of the parameters  $d_s$  and  $a$  are established for each specific case and depend on the nature and conditions of application of the tear-off force on a reinforced concrete structure element (through consoles, adjacent elements, etc.). Based on the given calculation, in accordance with the load-bearing capacity requirements, the required amount of transverse reinforcement is established in shear zone.

In technical code for established practices TKP EN 1992-1-1-2009 [2], in accordance with clause 9.2.5, in the case of one beam resting on another

beam in the area of the element intersection, installing additional reinforcement is required, which is designed to absorb the mutual reaction, while the supporting reinforcement is clamps, which embrace the longitudinal reinforcement of the support element, some of which may be located outside the volume of concrete which is common to the two beams (zone B, Figure 2). At the same time, the requirements for the diameter and spacing of clamps are not regulated in this normative document.



**A** – support beam with height  $h_1$ ;  
**B** – supported beam with height  $h_2$  ( $h_1 \geq h_2$ )

**Figure 2** – Placement of the support reinforcement in the area of the two beam intersection (plan view) [2]

Similar recommendations for the design of the junction of the main and secondary beams are contained in construction rules SP 63.13330.2018 which are current in the Russian Federation [clause 10.4.12, 4], which propose to install additional transverse reinforcement in the form of clamps which cover the longitudinal reinforcement to absorb the reaction from the secondary beam (Figure 3). In this case, in the main beam this reinforcement should be installed on a section  $b+2h$  in length, in the secondary beam – on a section  $h/3$  in length, where  $b$  and  $h$  are the width and height of the secondary beam.

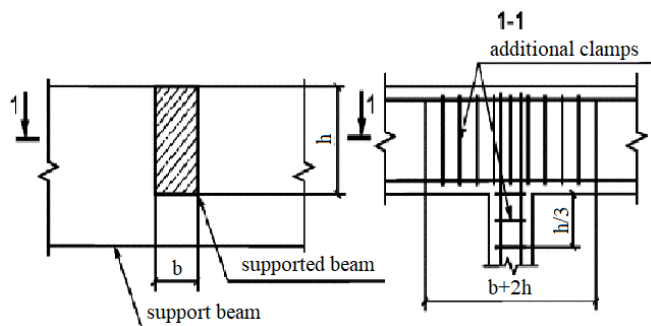


Figure 3 – Placement of support reinforcement in the area of the two beam intersection [4]

American standards ACI 318R-19 [5] envisage the reinforcement with clamps of only the support (main) beam to absorb the reaction from the supported (secondary) beam in the place of their junction as shown in Figure 4. Additional reinforcement in the beam junction zone may be absent if the following condition is fulfilled:

$$V \leq 3b_w d \sqrt{f'_c}, \quad (3)$$

where  $f'_c$  – the specified compressive concrete strength;

$b_w$  – width of the supported beam;

$d$  – the effective height of the supported beam.

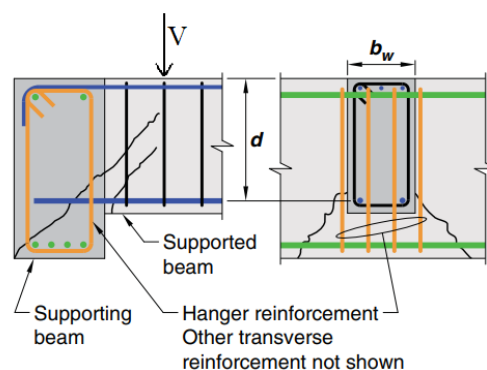


Figure 4 – Diagram of the support beam reinforcement in the junction zone with the supported beam [5]

Canadian Standard CSA A23.3-04 [6] contains more detailed recommendations for additional transverse reinforcement of a support beam to accommodate the force transmitted from the supported beam, provided that the bottom edge of the given beam is not lower than the bottom edge of the support beam. In this case, the tensile force absorbed by the additional reinforcement is:

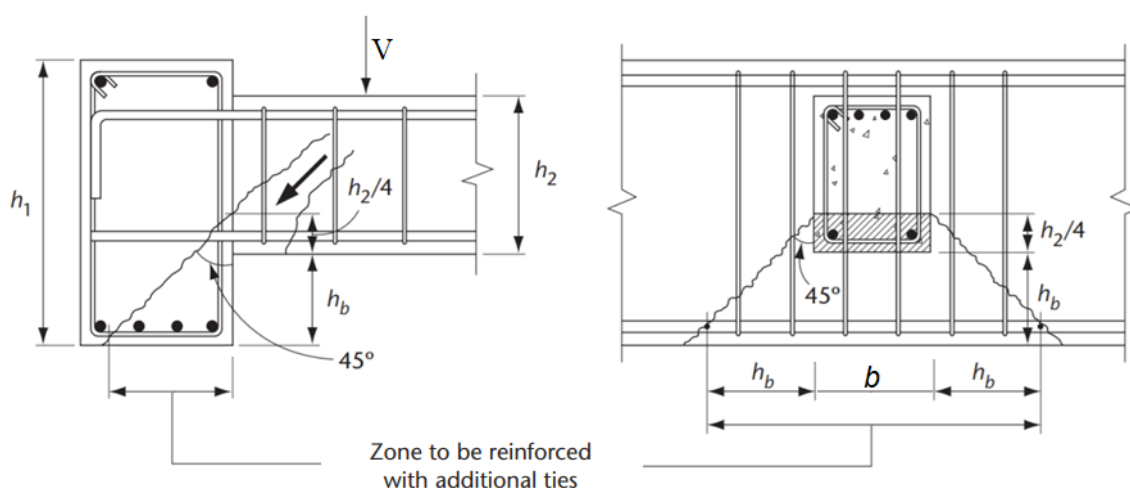
$$F = V \cdot \left( 1 - \frac{h_b}{h_1} \right), \quad (4)$$

where  $V$  – the transverse force on the prop of the support (main) beam (Figure 5);

$h_b$  – distance from the bottom edge of the supporting (main) beam to the bottom edge of the secondary beam;

$h_1$  – the supporting beam height.

Additional transverse reinforcement must be installed along the entire height of the support beam in such a way so as to cover the tear-off area formed by slanting cracks developing at an angle of  $45^\circ$  from the shear boundary located at a height  $\left(\frac{1}{4}\right) \cdot h_2$  from the bottom edge of the supported beam, to the bottom edge of the support beam.



$h_1$  – support beam height;  $h_2$  – supported beam height;  $h_b$  – distance from the bottom edge of the support beam to the bottom edge of the supported beam;  $b$  – supported beam width

Figure 5 – Diagram of the location of additional transverse reinforcement under local action of tensile strength [6]

According to Canadian standards, additional reinforcement of the support beam in the junction area with the supported beam is not required in cases where:

1. the shear boundary passes above the upper edge support beam;
2. the average tangential stresses at the shear boundary exceed

$$\tau \leq 0,23\lambda\varphi_c\sqrt{f'_c}, \quad (5)$$

where  $\lambda$  – a coefficient that takes into account the performance characteristics of concrete, the density of which does not exceed 1850 kg/m<sup>3</sup>;

$\varphi_c$  – coefficient of concrete working conditions.

For the purpose of analyzing the factors influencing the actual size of the area where additional transverse reinforcement is installed, which are taken into account in various standards, we will consider three options for pairing the support and supported beams:

- option I – the heights of the main and secondary beams are equal (Figure 6, a);
- option II – the height of the main beam is greater than the height of the secondary beam and their upper edges coincide or the secondary beam is located within the height of the section of the main beam (Figure 6, b);
- option III – the height of the main beam is greater than the height of the secondary beam and their lower edges coincide (Figure 6, c).

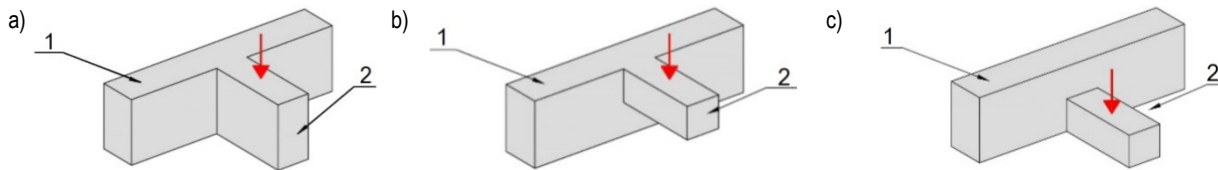


Figure 6 – Options for the location of the main and secondary beams relative to each other [7]

For each of the above options for the relative arrangement of beams in accordance with the requirements of various standards, Table 1 shows the values of the zone size for installing additional reinforcement.

Table 1 – Size of the area for installing additional transverse reinforcement

No.	Name of the regulatory document	Size of additional transverse reinforcement area		
		Option I	Option II	Option III
1	SP 5.03.01-2020 [1-1]	$b+2d_s$		$b+2d_s$ , где $d_s < h_2$
2	TKP EN 1992-1-1-2009 [1-2]	$h_1/2$		
3	SP 63.13330.2018 [1-3]	$b+2h_2$		
4	ACI 318R-19 [1-4]	$> b$		
5	CSAA23.3-04 [1-5]	$b+h_2/2$	$b+2(h_2/4+h_b)$	$b+h_2/2$

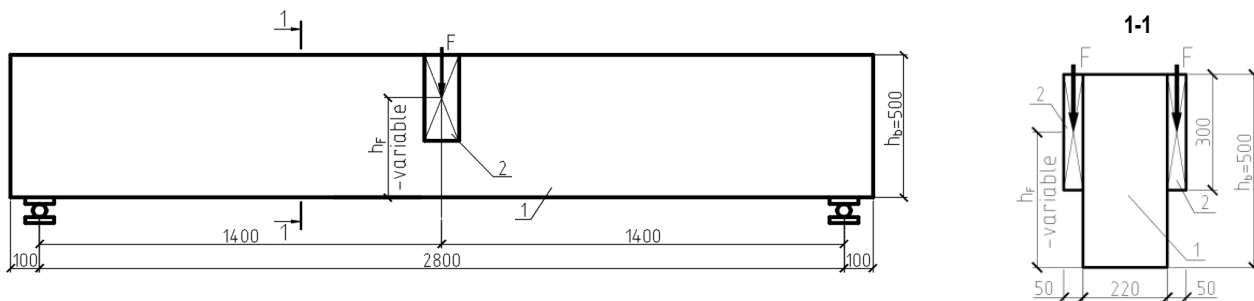
From the analysis of the data in Table 1, it was established that the size of the tear-off zone according to [2], [4] and [5] is not influenced by the relative position of the beams in height relative to each other, while the norms [1] and [6] take this feature into account. However, it is obvious that when calculating the tear-off load-bearing capacity, the location of the point of application of the tear-off force and the angle of inclination of the diagonal crack will be of no small importance.

It is also important to note that none of the considered regulatory documents takes into account the contribution of the concrete work to the tear-off resistance (tensile concrete work [8]), the influence of transverse reinforcement established on the basis of the general calculation of strength along slanting sections, pre-stressing forces, etc.

#### Numerical study of the stress-strain state of the tear-off zone under local action of tensile forces

To obtain data on the characteristics of the formation and development of cracks in the tear-off zone of reinforced concrete elements to which a local tensile force is applied, to reveal the dependence of the angle of inclination of the tear-off cracks and the size of the tear-off zone on the position of the point of application of the tear-off force, a numerical experiment was carried out using the "Abaqus/CAE" software complex, which implements finite element method [9].

Finite element analysis was carried out for a sample prepared for experimental research, the testing diagram and reinforcement of which are presented in the Figure 9.



1 – support beam; 2 – supported beam;  $h_1$  – height of the support beam;  $h_2$  – distance from the center of application of the tear-off force to the bottom edge of the support element

Figure 9 – Test diagram

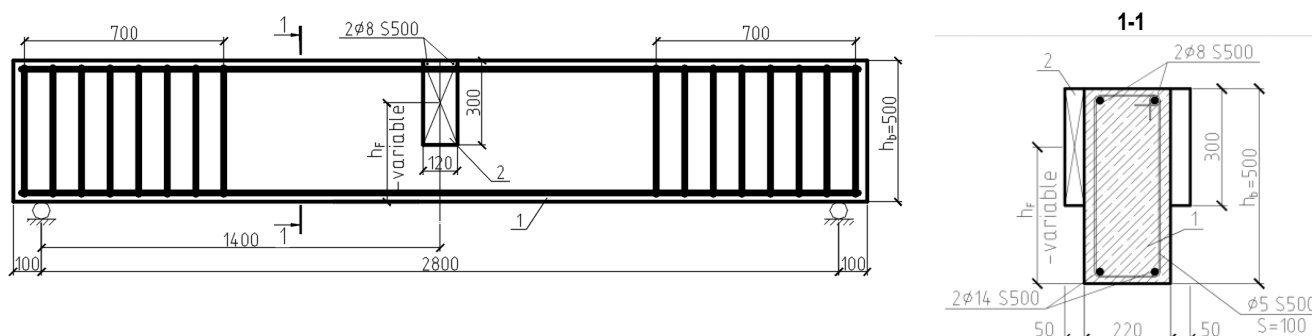


Figure 10 – General view of the tested junction

The support beam is designed to provide bending resistance for the calculated value of the impact effects  $F_u$  in accordance with [1].

In order to analyze the size of the tear-off zone in the case where the lifting load is perceived only by concrete, additional transverse reinforcement in the beam at the point where the tear-off force is transmitted, was not provided.

For numerical analysis, three diagrams of the junction of the supported and support beams (secondary and main) were simulated:

- I – the upper edges of the beams coincide ( $h_f / h_b = 0,7$ );
- II – the supported beam is located within the height of the support beam section ( $h_f / h_b = 0,5$ );
- III – the lower edges of the beams coincide ( $h_f / h_b = 0,36$ ).

Based on the results of numerical experiments, a picture of the stress-strain state in the separation zone and the distribution of tear-off cracks along the height of the section of the supporting element (main beam) were obtained for each of the coupling schemes (Figure 11).

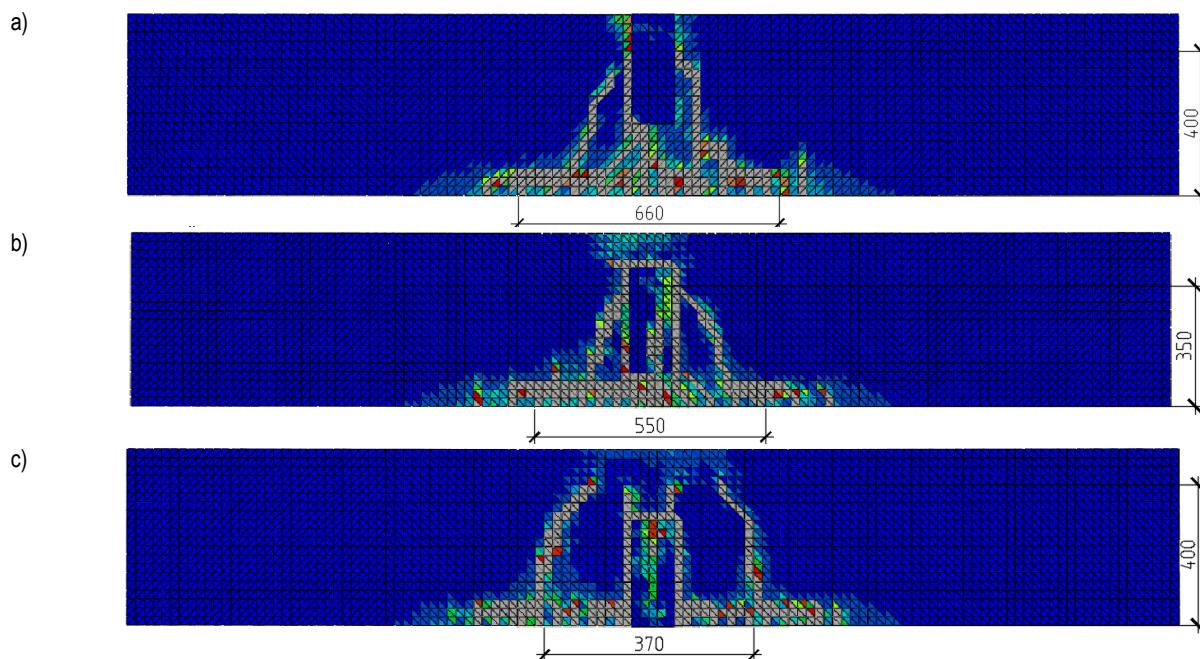


Figure 11 – Distribution of cracks in the tear-off zone of the simulated beams

As a result of a numerical analysis, it was established that for schemes I and II of the pairing of beams, the first normal cracks in the support beams appeared in the zone of action of the maximum bending moment on the outermost tensile fibers at a load of  $0,8F_{Ed,a1}$  and  $0,8F_{Ed,a2}$  ( $F_{Ed,a1}$  and  $F_{Ed,a2}$  is the limiting value of the tear-off force for the first and second beam pairing diagrams, respectively), which is about  $0,4F_u$ . Further development of normal vertical cracks did not occur due to the perception of acting tensile forces by the longitudinal reinforcement of the tensile zone of the supporting beam. With an increase in load to  $0,9F_{Ed,a1}$  and  $0,9F_{Ed,a2}$ , clearly defined diagonal cracks started in the tensile zone of concrete, oriented toward the center of application of the acting tear-off force. According to the results of the numerical experiment, the angle of inclination of the separation cracks for option I of the beam connection was  $36^\circ$  to the vertical (Figure 11, a), for option II –  $31^\circ$  (Figure 11, b), the

dimensions of the tear-off zone are: 400 mm height and 660 mm length of the base of the separation pyramid for option I pairing of beams, and 350 mm and 550 mm – for option II.

Unlike the first two beam pairing diagrams, the third scheme has a different nature of destruction. A local crack in the tensile zone with increasing load has a clearly expressed appearance of a crack normal to the longitudinal axis of the element to the level of the top of the adjacent beam (Figure 11, c). Above the level of the adjacent beam, the crack deviates at an angle of  $56^\circ$  in the direction of the axis of application of the local calculated strength. The destruction occurred at a load of  $0,85F_{Ed,a3}$  also as a result of the tear-off of a section of concrete in the form of a pyramid of 400 mm in height and a foundation of 370 mm in length.

The data obtained from the simulation results were compared with the sizes of the tear-off zones obtained by using the calculation methods given in the above-mentioned regulatory documents for each of the beam pairing options (Table 2).

**Table 2** – Calculation of the area for installing additional transverse reinforcement for numerical modeling samples

No.	Name of the regulatory document	Size of the additional transverse reinforcement area, mm		
		Option I	Option II	Option III
1	SP 5.03.01-2020 [1-1]	750*	550*	350*
2	TKP EN 1992-1-1-2009 [1-2]	≤500	≤500	≤500
3	SP 63.13330.2018 [1-3]	720	720	720
4	ACI 318R-19 [1-4]	≥120	≥120	≥120
5	CSAA23.3-04 [1-5]	670	370	270
6	Numerical model	660	550	370

Note. The center of gravity of the section of the supported beam is taken as the point of application of the tear-off force.

As noted earlier, the calculation methods given in foreign regulatory documents [2], [4] and [5] regulate the size of the tear-off zone (installation of additional reinforcement) without reference to the relative position of the beams. While the standards [1] and [6] take into account the influence of the position of the point of application of the tear-off force: when the point of application of the tear-off force moves to the lower edge of the support beam, a gradual decrease in the size of the tear-off zone occurs, and the destruction has a clearly marked character of destruction along the normal section.

Taking into account the fact that the additional transverse reinforcement of the tear zone, provided that  $d_s > 0$ , accounts for only part of the tear-off force, equal to  $F_{Ed} \cdot (1 - d_s/d)$  [1], therefore  $F_{Ed} \cdot d_s/d$  should be absorbed by the element without additional structural measures. In the process of numerical modeling of the operation of prototypes under the action of a tensile force applied locally, the inclusion of longitudinal reinforcement in the perception of tear-off force was established. From which it follows that when operating elements of reinforced concrete structures under the local action of tensile forces, it is necessary to take into account not only the contribution of the tensile concrete work and the influence of transverse reinforcement established on the basis of the general calculation of strength along slanting sections, but also the work of the longitudinal reinforcement of the element.

### Conclusions

1. Methods for calculating elements of reinforced concrete structures for tear-off, given in domestic and foreign regulatory documents of the near and far abroad, for the most part, contain general requirements for installing additional structural transverse reinforcement of the main beam, and, as a rule, do not take into account the value of the current tear-off force, and also the tear-off work of concrete (tension) and the influence of transverse reinforcement established on the basis of the general calculation of strength along slanting sections.

2. Analysis of the numerical research data showed that the value of the tear-off force depends on the point of application of the tear-off force relative to the height of the section of the support beam. The results obtained are consistent with condition (1) given in [1], according to which the force perceived by additional transverse reinforcement in the tear-off zone increases when the point of application of the tear-off force is shifted to the lower edge of the element.

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